Objectives

Adults Learning Mathematics (ALM) – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum that brings together researchers and practitioners in adult mathematics/numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members, an enormous contribution has been made to making available research and theories in a field which remains under-researched and under-theorized. In 2005, ALM launched an international journal dedicated to advancing the field of adult mathematics teaching and learning.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

- Research and theoretical perspectives in the area of adults learning mathematics/numeracy
- Debate on special issues in the area of adults learning mathematics/numeracy
- Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

The ALM International Journal is published twice a year.

ISSN 1744-1803


Editorial Team:

Javier Díez-Palomar, Universitat Autònoma de Barcelona, Spain
Jeff Evans, Middlesex University London, UK
Linda Galligan, University of Southern Queensland, Australia
Lynda Ginsburg, Rutgers University, USA
Graham Griffiths, University College London, UK

Editorial Board:

Marta Civil, University of Arizona, U.S.
Diana Coben, University of East Anglia, UK
Gail FitzSimons, University of Melbourne, Melbourne, Australia
Gelsa Knijnik, Universidade do Vale do Rio dos Sinos, Brazil
John O’Donoghue, University of Limerick, Ireland
Katherine Safford-Ramus, Saint Peter’s University, USA
Lieven Verschaffel, University of Leuven, Belgium
John Volmink, Natal University Development Foundation, Durban, South Africa
Tine Wedege, Malmö University, Malmö, Sweden
Editorial
Javier Díez-Palomar, Jeff Evans, Linda Galligan, Lynda Ginsburg and Graham Griffiths

Student perceptions of mathematics readiness from a university preparatory program to undergraduate studies
Clare Robinson, Linda Galligan, Zanubia Hussain, Shahab Abdullah, Anita Frederiks and Andrew Wandel

Proportional reasoning of adult students in a Second Chance School: The subconstructs of fractions
Aristoula Kontogianni and Konstantinos Tatsis

Safety First: Combining Task Models of Medical Devices with Numeracy Skills and Technical Competence
Judy Bowen and Diana Coben

Welcoming Articles about Practice to ALM-IJ
Lynda Ginsburg
This issue of Adults Learning Mathematics: An International Journal includes three articles and an invitation to submit Articles of Practice to the Journal. The articles present research situated in three specific environments in which adults learn mathematics: a school for adults who did not complete secondary school as teenagers, a program for adults preparing for a specific career, and initial tertiary studies in mathematics.

In the article, “Student perceptions of mathematics readiness from a university preparatory program to undergraduate studies,” Clare Robinson, Linda Galligan, Zanubia Hussain, Shahab Abdullah, Anidta Frederiks and Andrew Wandel provide an analysis of the relationship between students’ perceptions of their preparation for entry-level undergraduate mathematics and their success in those courses. Of particular interest were those students who entered university through a Tertiary Preparation Program (TPP) pathway rather than through traditional pathways such as completion of appropriate secondary or vocational school mathematics. The TPP students were more likely than other entering students to be low SES and older than 30 years. Following completion of their first college-level mathematics course, 688 students (with 14% TPP students) were surveyed and reported on how prepared they felt regarding 12 specific mathematics topics and for the course they had just completed. In addition, their course grades were compared with their perceptions. Findings indicated that completion of the TPP courses helped students feel prepared for their undergraduate courses, though there were some mismatches between the content of particular TPP courses and the expectations of specific undergraduate entry-level courses. The TPP courses did seem to ameliorate some of the expected negative effects of low SES and mature age. The authors note that the self-efficacy and self-confidence developed through such preparatory mathematics courses are particularly valuable for at-risk students, but attention must be paid to the content alignment between preparation courses and the undergraduate courses that the students will face next.

The article on "Proportional Reasoning of Adult Students in a Second Chance School: The Subconstructs of Fractions", by Aristoula Kontogianni and Konstantinos Tatsis, focuses on the learning of proportional reasoning among adult students at a Second Chance School in Greece. They combined the evaluation of answers to 13 open-ended tasks completed by 30 students, with transcripts of "clinical interviews” (e.g. Ginsburg, 2009) with five students. A particular strength of this work lies in the combining of the theory of subconstructs of fractions (Kieren, 1980) from mathematics education research, with data from empirical work by others and themselves, in adult mathematics education. Their "mixed methods" data revealed some difficulties related to specific fraction subconstructs among the sample as a whole, and some gaps in reasoning by particular students in the interviews. This paper sheds valuable light on adult mathematics practice, in that it offers pointers to areas for which additional emphasis in teaching and learning may be needed.

ALM-IJ has published a number of articles concerning issues in nursing numeracy over the years. In this issue, Bowen and Coben (“Safety First: Combining Task Models of Medical Devices with Numeracy Skills and Technical Competence”) break new ground by aligning cross-disciplinary work in technology design (specifically syringe drivers which can have user error) and numeracy education using a Competence Model for medication dosage calculation problem-solving (MDC-PS). They compared the description and analysis of two task models. The first task model was with the device itself: the step-by-step sequences of the syringe driver mechanism. The second task model was the step-by-step sequence of setting up the actual syringe driver and then using it by a nurse using MDC-PS. The authors were then able to identify a relationship between these two domains, where fulfilling...
tasks described in the device models depended on competencies outlined in the MDC-PS. It also identified potential user errors that could then be used to inform both device design and user education.

The final contribution in this issue announces and explains the inclusion of a new section of the Journal, “Articles about Practice.” The presence and contributions of adult education practitioners have always been evident at the annual Adults Learning Mathematics conferences, so it seems reasonable that another opportunity for the larger adult education mathematics community to learn from and about practitioners’ experiences and dilemmas should be made available. The article describes the rationale for such articles, suggests a range of topics, and proposes a general structure for submissions.

Beginning with this issue, the editorial duties are being shared among Jeff Evans, Linda Galligan, Lynda Ginsburg, and Graham Griffiths, with guidance and wisdom from the former Chief Editor, Javier Díez-Palomar. We hope you will find the articles in this issue interesting and thought-provoking, and we encourage you to submit research or practice articles to the ALM International Journal.

The Editorial Team
Student Perceptions of Mathematics Readiness from a University Preparatory Program to Undergraduate Studies

Clare Robinson
Linda Galligan
Zanobia Hussain
Shahab Abdullah
Anita Frederiks
Andrew Wandel

University of Southern Queensland, Australia
<Clare.robinson@usq.edu.au>

Abstract

For many mature-aged students, there can be a considerable gap between finishing formal study and beginning university. They are often required to draw on qualifications and skills, including mathematics, which are dated, but necessary for success. Most universities offer several pathways that prospective students can take to enter university: from school; from vocational education; based on prior learning; or via a university preparatory program. Even with extra preparation from a preparatory program, success is not guaranteed, as the assumed skills can be quite specific to particular degrees.

Mathematics prerequisites for entry into undergraduate programs are not enforced in many Australian universities. Students are only advised of the level of senior mathematics that is ‘assumed’ or ‘recommended knowledge’ for their degree and are often uncertain if their current mathematical skills are enough to understand the content of a particular course.

A survey to investigate students’ perceptions of how well their pre-university studies prepared them for first year undergraduate courses that required quantitative skills was conducted at one regional university in Australia. As part of this survey, the perceptions of mathematical preparedness of students who had qualified for entrance to undergraduate studies via the university’s tertiary preparation program were investigated. Most respondents who transitioned through the preparatory program felt well prepared for their undergraduate mathematics. However, a greater proportion of these students either withdrew from the undergraduate quantitative course that they had enrolled in or achieved lower grades when compared to other survey respondents who had entered university through more traditional pathways.

Keywords: mathematics; tertiary preparation; student perceptions; enabling; pre-requisites; STEM

Introduction

Worldwide, there is government and industry impetus to lift the overall scientific literacy of their populations, as there is a strong link between nations with leading economies and strong science education and research systems (Marginson, Tytler, Freeman, & Roberts, 2013). In many English-speaking countries, including the United States (Atkinson & Mayo, 2010), the United Kingdom (Morgan & Kirby, 2016) and Australia, there is concern over the declining trends in Science, Technology, Engineering and Mathematics (STEM) participation, and the longer tail of low-achievers at school level compared to many Western European and East Asian countries (Wright, 2010).
At the university level, mathematics lecturers have noted the increase in the number of students who lack the appropriate mathematical background to cope with first year science (Poladian & Nicholas, 2013a; Rylands & Coady, 2009) and mathematics subjects (Vandenbussche, Ritter, & Scherrer, 2018). Many universities around the world attempt to ameliorate the difficulties encountered by mathematically under-prepared students by offering mathematics courses within various programs, often referred to as enabling, preparatory, bridging or remediation programs. The aim of these courses is to allow students to obtain prerequisite or assumed knowledge in mathematics before commencing their degree program (Croft, Harrison, & Robinson, 2009; Greefrath, Koepf, & Neugebauer, 2017; MacGillivray, 2009). Successful completion of a mathematics component in such programs has been shown to be a predictor for engagement and success in further university mathematics studies (Varsavsky, 2010).

Mathematics is a major part of many Australian university preparatory programs (Irwin, Baker, & Carter, 2018). The Tertiary Preparation Program (TPP) at the University of Southern Queensland (USQ) is one of these preparatory programs. It is a fee free program with an open entry policy and provides students with a direct entry pathway to an undergraduate degree at the university. Approximately 10% of students enter their undergraduate degree at USQ through the TPP. Determining whether there is a successful transition of this group of students into their undergraduate studies is therefore of importance to the university.

Research conducted in 2013 on students’ perceptions of their preparedness in first year undergraduate mathematics at USQ (Dalby et al., 2013), found a significant number of students in the science based degrees acknowledged that they were inadequately prepared. Further investigation particularly around STEM related degrees was conducted in 2015 (Galligan et al., 2017). As part of this research, those students who had completed the TPP were identified. The focus on these students had two aims. The first was to determine whether TPP respondents in the study perceived themselves to be well prepared for their undergraduate mathematics, and to identify particular mathematics topics in which these students felt underprepared. The second was to compare their perception of preparedness and their overall achievement in their undergraduate course, with that of the other respondents who had entered university through more traditional pathways.

Literature Review

This section provides a background to this paper, discussing the reasons behind the decline in mathematics and science participation and success at university level in Australia, factors affecting degree completion as a whole, and the role of university preparatory/enabling programs.

Decline in mathematics and science participation in Australia

An Australian government report (Office of the Chief Scientist, 2013p. 13) notes that despite an increase of nearly 30% in science enrolments in higher education since 2007, student enrolments in key fields such as chemistry, physics and mathematics drop steeply after first year. This could be linked to a decline of student enrolment in challenging mathematics and science based subjects at school level (Education Council, 2018; Marginson et al., 2013). There has also been a trend for school students to take lower levels of school mathematics, attributed to several factors including the lowering of prerequisites by universities in order to attract more enrolments (Wright, 2010). Studies have also determined that the level of secondary mathematics completed and quality of achievement at that level is one of the factors associated with successful/unsuccessful outcomes in first year mathematics courses (Greefrath et al., 2017; Peard, 2004; Universities Australia, 2013; Wright, 2010).

There has been concern that students transitioning to university from the vocational education sector may not have the mathematical background to cope with science based degrees (King, Dowling, & Godfrey, 2011; Penesis et al., 2015). Another study found that students who entered engineering
degrees on the basis of vocational education studies or on mature\textsuperscript{1} age or special entry criteria had a far lower degree completion rate than the rest of their cohort (King et al., 2011).

**Factors affecting degree completion**

Many factors can affect the completion of a degree. A recent Australian Grattan Institute report identified some of these factors as academic ability, motivation, persistence, time put into study, study practices, financial support, social support, academic support and teaching quality (Norton, Cherastidtham, & Mackey, 2018). This report singles out mature aged students as being particularly vulnerable to dropping out. Being mature aged and from a low SES background compound the difficulties with academic integration (Tones, Fraser, Elder, & White, 2009).

Regional universities in Australia have a more diverse cohort in terms of age, SES, and academic experience and have lower completion rates in comparison to metropolitan universities (Australian Government, 2017; Norton et al., 2018). In terms of STEM, National benchmarking data also indicate a drop in the level of achievement of students in school mathematics and science in regional and rural areas of Australia compared to metropolitan areas (Panizzon & Pegg, 2007). Additionally, low SES students are four times as likely to be low performers in mathematics compared to high SES students (Thomson, 2016).

**University preparatory programs in Australia**

In an extensive review of university preparatory programs in Australia, it was found that these programs primarily provide alternative pathways to university for non-traditional students, with the intent to address the outcomes of disadvantage (Bird & Morgan, 2003; MacGillivray, 2009). Non-traditional students belong to one or more of the following groups: low socio-economic background, regional and remote areas, people with a disability, Aboriginal and Torres Strait Islanders and Non-English speaking backgrounds. The Australian government has recognised the importance of preparatory programs in the provision of both general study skills and discipline specific knowledge to students with social or economic disadvantage (Australian Government, 2017). In 2010, the Australian government stated the aim to increase the percentage of university students from low socio-economic status (SES) background from 16\% to 20\% by 2020 (Bradley, 2008). As a result, one of the proposals put forward by Universities Australia (Universities Australia, 2013) was for Australian universities to broaden pathways into university, and for the government to maintain equity programs to improve access for students from low SES backgrounds. Low-SES students are overrepresented in regional universities (Network Regional Universities, 2013) and therefore these universities have a higher percentage of total student load from preparatory courses (Pitman et al., 2016). This research also noted that students who had entered university via a preparatory pathway were below the national average in terms of retention and success in their subsequent undergraduate studies (Pitman et al., 2016).

Students enrolling in preparatory programs may have many of the recognised difficulties associated with adjusting to university, including having the key skills to succeed. One key skill is the required mathematics level for STEM programs (Rylands & Coady, 2009). Hence, there is a need to assess whether these students have attained key mathematics skills within their preparatory courses. While some data can be collected through the use of student evaluation and undergraduate performance (Shah & Whannell, 2016), asking students their perceptions of the attainment of these skills is important, as self-efficacy and confidence are known to be indicators of success in mathematics (Ernest, 2003; Parsons, Croft, & Harrison, 2009).

\textsuperscript{1} Defined as 25 or over
Method

For this paper, we drew on data from students enrolled in Semesters 1 and 2 in 2015. Ethics clearance was obtained to survey and communicate with students enrolled in thirteen STEM related first year courses (two physics course, one chemistry course, one biology course, three engineering courses, two nursing courses, two engineering mathematics courses, one other mathematics course and one standard statistics course). All the examiners (coordinators) of these key first year courses within the university gave permission for the researchers to email their students with an invitation to fill out a survey that investigated the respondent’s perception of preparedness for the mathematics based course undertaken in the previous semester. Students were encouraged to participate by offering them the chance to win a $100 book voucher. The survey was emailed to the students after the semester results had been released (including students who had dropped the course) and elicited responses from 688 students over the two semesters (with 93 respondents having previously completed a TPP mathematics course). The response rate was 10.9 % of the total number of students enrolled in the selected courses. A significant number of students also agreed to be interviewed, but once approached, were less responsive.

The survey questions sought students’ perceptions of their preparation in various mathematics topics using a Likert scale (see Appendix). The topics included calculator use, decimals, percentages, ratio, algebra, statistics, problem solving, trigonometry, and calculus. Students were also asked if they considered their overall mathematical preparation to have been adequate for the course in question. There were a number of open-ended questions to explore other factors that students may have felt had contributed to their success or failure. Qualitative data from the relevant questions were analysed in NVivo using constant/comparative method (King et al., 2011). Some attempt was made to capture the conceptual as well as the thematic regularities in the data but most of the answers were too terse to be particularly useful in this regard. The grade achieved by each respondent in his or her undergraduate mathematics course was also recorded. Basic demographic information was also collected.

The survey responses of students that identified as having entered their undergraduate degree via the TPP were analysed according to their exit level of TPP mathematics. The level of mathematics completed within the TPP depends upon the student’s intended degree. Table 1 lists the prerequisite level of TPP mathematics required for various degrees, an outline of the topics covered in each course and the approximate secondary school equivalent. There are three university entrance levels of final year mathematics in Queensland, with Mathematics C being the most challenging. None of the mathematics courses in this survey had Mathematics C as a prerequisite. Although an approximate equivalency has been drawn between TPP mathematics and Queensland school mathematics, the time frame of the former courses (one semester) does not allow for the same breadth and depth of study of mathematics topics as in school.

Results

Demographics of the participants

For this paper, the 688 survey participants were divided into the following two groups for comparison:

- Ex-TPP respondents: those students that had entered their undergraduate studies via the TPP (14% of total respondents); and
- Other pathway respondents: those students that entered their undergraduate studies having completed a level of year 12 mathematics (71% of total respondents), those respondents that only formally completed Year 9 or 10 at school (9%), vocational mathematics (2%) or did not indicate their last level of mathematics (4%).
The second group includes students who stated that their last formal mathematics study was year 9 or year 10 school mathematics. As these students had not needed to transition through the TPP to gain university entrance, it is assumed that their overall qualifications were deemed sufficient for their degree of enrolment.

In 2015, 23.4% of all USQ students were identified as low SES compared to 36.3% of TPP students. In this survey, 35.6% of students who had entered university via the TPP were identified as low SES, which suggests that they were a representative sample in terms of overall TPP pathway enrolments at this university.

Figure 1 shows the ages of respondents to the survey according to university entry pathway. The percentage of ex-TPP respondents over 30 years of age (57%) is greater than that of other respondents (43%). Only 8% of ex-TPP students were below 21 years of age compared to other respondents (24%). The high percentage of mature-aged and low-SES students in the ex-TPP respondents suggest that some of these students could be at greater risk of dropping or failing their mathematical course compared to other respondents (Thomson, 2016; Tones et al., 2009).

**Ex-TPP student perceptions of preparedness according to mathematics topic.**

Students were asked to select their level of preparedness for various mathematical topics using categories of “Well prepared”, “Prepared”, “Poorly prepared” and “Very poorly prepared”. The last two categories were combined due to the relatively small number of responses. The topics were Calculator Use, Decimals, Fractions, Percentages, Graphs, Ratios, Basic Algebra (e.g. using a formula), Advanced Algebra (e.g. rearranging equations, factorising), Problem Solving (e.g. interpreting and solving word problems), Statistics, Trigonometry (e.g. sine, cosine, tangent) and Calculus (e.g. basic differentiation and integration). Figures 2 to 4 reflect perceptions of preparedness according to level of previous TPP mathematics completion. From Figure 2 it can be seen that although generally confident in areas of arithmetic, a high proportion of students that had completed TPP-Maths 1 indicated feeling underprepared for statistics and what they perceived to be “advanced” algebra (over 40%). Thirty percent of students also felt underprepared for problem solving. As this course does not cover calculus,
the few students that attempted undergraduate courses with a calculus component naturally felt underprepared.

![Figure 1: Comparison of age distribution of other entry pathway respondents (n=567) and ex-TTP respondents (n=93)](image)

*Figure 1. Comparison of age distribution of other entry pathway respondents (n=567) and ex-TTP respondents (n=93)*

![Figure 2: Perceptions of preparedness of ex-TTP-Maths 1 students (45)](image)

*Figure 2. Perceptions of preparedness of ex-TTP-Maths 1 students (45)*

Figure 3 shows that most students felt underprepared for their initial encounter with calculus. This is not surprising as this topic is not covered in TPP-Maths 2. Over 20% of students felt underprepared for the problem solving and statistics in their course. The emphasis in TPP-Maths 2 is on trigonometry rather than statistics, for students that plan to enrol in engineering degrees. All students felt prepared for basic algebra, while approximately 20% indicated a problem with advanced algebra.
In contrast to the confidence with algebra shown by ex-TPP-Maths 2 students, Figure 4 reflects that over 20% of students felt poorly prepared for the advanced algebra in their course after having completed TPP-Maths 3. These students would have enrolled in higher-level undergraduate courses with more challenging algebra. As with TPP-Maths 2, the emphasis is on trigonometry rather than statistics for students that plan to enrol in engineering degrees. This is reflected in the 20% who felt underprepared for statistics.

Topics that show high percentages of students that felt underprepared give an indication as to where the relevant TPP mathematics courses should be reviewed, from a student’s perspective.

**Ex-TPP student perceptions of overall preparedness for undergraduate mathematics**

In the survey, students were asked to select a response to the statement “Overall, my mathematics preparation was adequate for this first year course that I studied” with either “Strongly” Agree”, “Agree”, “Neutral”, “Disagree” or “Strongly Disagree”. Although the majority of ex-TPP students felt prepared (71%), there were 16% of respondents that felt under-prepared and 13% that felt neither well nor poorly prepared. Figure 5 compares ex-TPP student responses to this question with their final grade achieved in their undergraduate course. Ten students perceived themselves to be underprepared despite
having passed their course. Most of those that did not complete their course also felt well prepared, but the survey did not identify their reasons for not completing.

![Figure 5. Ex-TPP students’ perception of preparedness compared to final course grade](image)

**Entry level into undergraduate mathematics courses**

There are no official figures as to how many TPP students do not complete the required level of preparatory mathematics (as listed in the University handbook) before enrolling in undergraduate mathematics courses. As shown in Table 2, the results from this survey reflect that this may be an issue, where 24% of ex-TPP Maths 1 respondents and 20% of ex-TPP-Maths 2 respondents attempted undergraduate mathematics courses without having completed the prerequisite TPP mathematics course. Of the eleven ex-TPP Maths 1 students in this category, six either failed or did not complete their undergraduate course. However, of the five ex-TPP Maths 2 students, four continued to pass their undergraduate course. These students may have increased the proportion of students in the survey who felt underprepared for their undergraduate mathematics.

**Table 2. Percentage of ex-TPP respondents enrolling in undergraduate mathematics courses without the correct prerequisite level of TPP mathematics**

<table>
<thead>
<tr>
<th>TPP mathematics exit level</th>
<th>Percent of exit level that did not complete prerequisite course</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPP-Maths 1</td>
<td>11 / 45 (24%)</td>
</tr>
<tr>
<td>TPP-Maths 2</td>
<td>5 / 25 (20%)</td>
</tr>
</tbody>
</table>

Responses by ex-TPP students to an open-ended question in the survey are shown in Table 3. The level of TPP mathematics previously completed and their undergraduate course have been included to add perspective. Apart from positive opinions about the TPP pathway, some of these responses reflect concern and uncertainty about the lack of clarity of mathematical pre-requisites.
Table 3. Student responses to the question “Are there any other courses that you wish to comment on with regard to your mathematics preparation?”

<table>
<thead>
<tr>
<th>TPP level</th>
<th>Maths level</th>
<th>Comment</th>
<th>Course studied</th>
<th>Course Grade achieved</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPP-Maths 1</td>
<td>Both the TPP courses [maths and non-maths] were invaluable to preparing me for University. I highly recommend them both especially if you haven't studied in a while.</td>
<td>Nursing Maths</td>
<td>Distinction</td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 1</td>
<td>Have more advertising for students about TPP so they know they have a program to assist them before they begin a course.</td>
<td>Biophysical Sciences in Health</td>
<td>Distinction</td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 1</td>
<td>The TPP course prepared me well. So I thought.</td>
<td>Introductory Engineering Mathematics</td>
<td>Fail*</td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 1</td>
<td>[Physics] should have a Maths pre-requisite course, I only had the TPP Maths course for preparation which was very basic compared to the requirements of this course.</td>
<td>Physics 1</td>
<td>Pass*</td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 2</td>
<td>I studied the TPP Maths 2 unit from the Tertiary Preparation Program at USQ prior to commencing my studies of the Spatial Science degree in semester 2. Since starting mathematics studies at USQ I have had nothing but positive experiences in the subject. Therefore I would not change anything.</td>
<td>Introductory Engineering Mathematics</td>
<td>Distinction</td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 2</td>
<td>As a mature student if I had not completed TPP Maths 1 and TPP Maths 2 I would have been completely lost with ENG1002 Introductory Engineering and Spatial Science applications</td>
<td>Pass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 2</td>
<td>The TPP was a well-structured program which helped me greatly in Semester 1 with ENM1500 Introductory Engineering Mathematics</td>
<td>Distinction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 2</td>
<td>ENG1002 may be better as not an introductory course as it assumes a lot of background knowledge of the subject. … there could be stipulations for the required amount of prior engineering mathematics needed to enter.</td>
<td>Introductory Engineering and Spatial Science applications</td>
<td>Pass</td>
<td></td>
</tr>
<tr>
<td>TPP-Maths 2</td>
<td>PHY1104 required some high level mathematics. I know of several other students who found it too difficult and were not prepared. I believe a prerequisite of Maths B or the equivalent TPP course should be applied.</td>
<td>Physics 1</td>
<td>Distinction</td>
<td></td>
</tr>
</tbody>
</table>

* Comments were from students that entered their degree with the wrong prerequisite level of preparatory mathematics

Comparison of ex-TPP respondents to other entry pathway respondents

Using the response to the question about overall perception of preparedness, respondents were compared across all pre-university mathematics attainment groups. Figure 6 shows the percentage of students within each pre-university mathematics group that felt prepared/poorly prepared for their undergraduate course. The majority of respondents that had either completed 12 years of schooling² (last 3 columns) or a TPP mathematics course (first 3 columns) felt prepared for the course in which

² Maths A = non-calculus based mathematics; Maths B = calculus based mathematics; Maths C = advanced mathematics
they were enrolled. The lower the students’ school exit level, the greater the proportion of students that felt underprepared for the mathematics encountered in their degree. This was also reflected in the responses given by students who had completed some mathematics in a vocational program. The proportion of TPP students that felt well prepared is comparable to those students that entered university with Queensland Year 12 Mathematics B (this includes basic calculus). Those students that did not indicate their pre-university mathematics attainment were not included in Figure 6.

![Figure 6. Perception of preparedness of respondents by entry to university pathway (n = 661)](image)

Although the proportion of ex-TPP students that perceived themselves to be well prepared for their undergraduate mathematics was relatively high, this does not reflect their actual achievement (76% pass rate) when compared to the other respondents (84% pass rate). Figure 7 compares the final grade results for the undergraduate course identified in the survey for the two groups. To simplify results, a grade of 75% or higher is classified as a “Distinction” and a grade between 50% and 74% is classified as a “Pass”. Figure 7 shows that although the majority of students in both cohorts passed their course, only 34.5% of ex-TPP respondents achieved a distinction in their course compared to 45% of the other students. The proportion of ex-TPP students that did not complete their course is approximately double that of other university entrance pathways.
Discussion

The main purpose of this study was to investigate students’ perceptions of their preparedness for the quantitative skills of their first year undergraduate mathematics courses. For this paper, the focus was on students who entered university after completing a pre–undergraduate university preparatory course (TPP).

Perception of preparedness of quantitative skills

This study explored students’ perceptions of preparedness for: calculator use, decimals, fractions, percentages, graphs, ratios, basic algebra, advanced algebra, problem solving, statistics, trigonometry, and calculus. Identified mathematical topics, where a large percentage of respondents that had completed TPP-Maths 1 felt underprepared, were statistics and advanced algebra (Figure 2). This may not necessarily be due to the cognitive skills of algebra and statistics, but non-cognitive skills as the latter have been known to affect student results. These include university expectations, preconception of the degree of difficulty of algebra (Tariq & Durrani, 2012), anxiety (Ashcraft, 2002), beliefs of the usefulness of this topic (van der Wal, Bakker, & Drijvers, 2017), and unfamiliar writing and language related to the understanding of mathematics and statistics (Dunn, Carey, Richardson, & McDonald, 2016). These factors should be taken into account when designing content of preparatory mathematics programs.

Enabling Progression

Overall, preparatory courses are similar to the final year of high schooling (Shah & Whannell, 2016) but it is important to note that each TPP mathematics course in this study is geared towards a specific degree. This means that the mathematical topics covered in each course are different. Hence, as expected, different levels of ex-TPP students showed different levels of preparedness for these concepts as reflected in the results section. In this study, the majority of students that completed a TPP mathematics course felt well prepared for their undergraduate mathematics course. Unsolicited comments about the preparatory program reflect previous research that indicates that students who enrol in mathematics preparatory courses value these courses not only as a means of upskilling their
mathematics but as a means to facilitate their transition to university (Gordon & Nicholas, 2013). In addition, preparatory or bridging programs can serve as a “vehicle to nurture the student’s ‘will to learn’” (Poladian & Nicholas, 2013b).

To be noted, students in preparatory programs may on occasions undertake undergraduate courses with a high level of mathematical content without having completed the prerequisite level of preparatory mathematics and hence do not have the appropriate skill set required for the course. Students need greater clarity on quantitative expectations within an undergraduate course. This clarity is necessary at the preparatory and undergraduate program level. For example, from informal discussions, several students stated they dropped one physics course due to their lack of appropriate level of mathematics skills assumed within the course. We can speculate that this happened for other courses, but further research would need to be conducted to identify students’ reasons for dropping mathematics related courses.

The issue of skill mismatch can be addressed in four ways. First, administratively, by automatically alerting such students to a possible mismatch between their current skills and the requirements of the degree. Second, in the preparatory program, by alerting students at the finalisation of their program or mathematics course to highlight possible mismatches, and thirdly in the degree course where there is quantitative content, by alerting students early to the numeracy demands of the course and offering pathways to overcome any gaps in knowledge. Finally, as implemented by other universities (e.g. University of Sydney) and recommended in reports, (Australian Academy of Science, 2016; Marginson et al., 2013) prerequisites could be reintroduced more broadly.

**TPP versus other entry pathways perceptions and achievement**

The proportion of ex-TPP respondents that indicated they were prepared for their undergraduate mathematical course was greater than those that had completed the equivalent of Queensland Mathematics A (non-calculus) or lower. However, ex-TPP respondents’ final course grades indicate that they generally achieve lower grades and are at a higher risk of not completing their mathematics based course than other entry pathway respondents as suggested in the literature (Poladian & Nicholas, 2013b). This group of ex-TPP respondents had a high proportion of low SES students (at 35.6%) and a high mature aged proportion (at 78%). Given that research (Norton et al., 2018) suggests low SES and mature age are factors in progression and retention, the ex-TPP students’ overall success rate of 76% is reasonable when compared to the 84% success rate of other entry pathway students’ in this survey.

**Conclusion**

This research contributes to tertiary education literature by probing the effectiveness of tertiary preparatory mathematics courses through the lens of students’ perceptions. Results from a survey on mathematical preparedness for first year undergraduate mathematics showed that approximately 70% of respondents who had completed TPP at the University of Southern Queensland in Australia felt well prepared for their undergraduate mathematical course studies. This is comparable to those respondents who had previously completed a high level of school mathematics, and is a good indication that completion of a university preparatory mathematics course had engendered self-efficacy and self-confidence in their mathematical ability. However, the survey also indicated that students in this group may be at greater risk of either failing or dropping out from their course compared to students who enter tertiary studies via other tertiary entrance pathways.

Conclusions drawn from this survey are limited to one regional university and may not reflect student perceptions or outcomes in other preparatory programs. Whilst these conclusions may be biased as the survey responses were voluntary, with only a small number of respondents having previously completed the TPP, this research provides some evidence that tertiary preparatory students are
Robinson, Galligan, Hussain, Abdullah, Frederiks, & Wandel. Student Perceptions of Mathematics Readiness from a University Preparation Program to Undergraduate Studies.

transitioning successfully into their undergraduate mathematics and perceive themselves to be well prepared for these courses.

Further research is needed to investigate if preparatory mathematics courses act as a critical filter to a student’s tertiary and later career aspirations (Shapka, Domene, & Keating, 2006). Although people working in the enabling sector share similar ideas about the mathematical requirements for academic preparation (Irwin et al., 2018), currently Australian university preparatory programs are not regulated or subject to external accreditation (Shah & Whannell, 2016). This study can feed into discussion of student attrition, success, and enabling provision (Department of Education and Training, 2017) and a push from some sectors to introduce clear prerequisite mathematics statements (Powell, 2018).

References


Appendix

Survey questions:

1. If you wish to take part in the draw for a $100 book voucher, please put in your email address.
   ________________________________

2. The degree and major that I am enrolled is __________________________
   For example BSc (Human Biology)

3. My mathematics preparation was (select one of the following)
   a) Tertiary Preparation Program (TPP)  TPP7181  TPP7182  TPP7183
   b) High school
      Year 9 or below
      Year 10
      Year 12 Maths A (basic)
      Year 12 Maths B (including calculus)
      Year 12 Maths C (more advanced than Maths B)
   c) Other, please say what ________________________________

4. How long ago did you last study mathematics?
   0 - 2 years     3 - 5 years     5 – 10 years     more than 10 years

5. Which mathematics based course are the following answers referring to? For example MAT1000, MAT1008
   ________________________________

6. In each of the following areas of mathematics, please select how well prepared you were prior to the start of the course listed in Question 5.

   Calculator Use

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

   Decimals

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

   Fractions

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

   Percentages

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

   Using graphs

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>
Robinson, Galligan, Hussain, Abdullah, Frederiks, & Wandel. Student Perceptions of Mathematics Readiness from a University Preparation Program to Undergraduate Studies.

**Ratios**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Basic Algebra (e.g. using a formula)**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Advanced Algebra (e.g. rearranging equations, factorising)**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Statistics**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Trigonometry (e.g. sine, cosine and tangent)**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Calculus (e.g. basic differentiation and integration)**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Problem solving (e.g. interpreting and solving word problems)**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

**Other maths topic**

<table>
<thead>
<tr>
<th>Well prepared</th>
<th>Prepared</th>
<th>Poorly prepared</th>
<th>Very poorly prepared</th>
<th>Not applicable</th>
</tr>
</thead>
</table>

7. Overall, my mathematics preparation was adequate for this first year course that I studied.

   Strongly agree  Agree  Neutral  Disagree  Strongly disagree

8. a) Are there any other courses that you wish to comment on with regard to your mathematics preparation?

   b) Do you have any comments?

9. Please briefly express your views on how the University could better prepare students for the mathematics they will see in first year.

10. Are you willing to help us by allowing us to interview you? We wish to try to obtain more detailed information about your experiences.  Yes / No

    If yes, please supply your email address.
Proportional Reasoning of Adult Students in a Second Chance School: The Subconstructs of Fractions

Aristoula Kontogianni
University of Ioannina-Greece
<desmath@gmail.com>

Konstantinos Tatsis
University of Ioannina-Greece
<ktatsis@uoi.gr>

Abstract
One of the subjects that constitute a problem for adult students of mathematics is the understanding of fractions and proportions. These notions are interrelated with proportional reasoning which was the focal point of our study. Using a mixed methods approach, we tried to assess the proportional reasoning of adult students during problem solving. We designed a task sheet with 13 open-ended tasks, each attempted by 30 adult students, and we conducted “clinical interviews” (Ginsburg, 2009) with five participants in order to acquire more data about their ways of thinking. For the task categorisation we used a typology of fraction subconstructs (Kieren, 1980). The results indicate that the adult students’ performance was relatively good despite their limited mathematical background. However, they had difficulties with the tasks that required a higher level of thinking.

Keywords: second chance school; proportional reasoning; rational numbers; fractions.

Introduction – Rationale of the study
In an era of rapid technological changes, the development of mathematical understanding in accordance with the contextual knowledge and skills for the use of mathematics in different settings is of great significance (FitzSimons, 2019). One of the most important understandings of mathematics is proportional reasoning, since it is strongly connected with other areas of science and mathematics across all levels of compulsory education. Proportional reasoning refers to the ability to use ratios in situations involving comparison of quantities and it has been described as the foundation for the understanding of algebra and the transition from informal to formal mathematical thought (Doyle, Dias, Kennis, Czarnocha, & Baker, 2016). Additionally, proportional reasoning “is one of the most commonly used applications of mathematics in everyday life” (Hilton, Hilton, Dole, & Goos, 2016, p. 194). Among the skills needed for proportional reasoning are multiplicative and relational thinking and an understanding of concepts such as rational numbers, fractions, decimals, multiplication, and division (Lamon, 2005). The comprehension of rational numbers and fractions is of especial importance since these notions are connected with real world situations (Behr et al., 1983).

While the importance of proportional reasoning is indisputable, many adults and students fail to reason proportionally (Lamon, 2007; 2012). This may have a serious impact for adults that have not completed compulsory education or lack basic mathematical knowledge, since they will not be able to operate in their daily lives. Based on the above assumptions we designed our study which aimed at the assessment of adults’ proportional reasoning in the setting of a Second Chance School in Greece, which offers education to adults over 18 years of age.
Proportional reasoning presupposes the understanding of fractions. The teaching of fractions is a precondition of the teaching of rational numbers. Whilst there are many studies about children's understanding of fractions and proportional reasoning (e.g. Charalambous & Pitta-Pantazi, 2007), few studies refer to adults. The objective of our study was to examine the ways adult students use fractions in proportional reasoning problems. Our main research question was: *To what extent do students understand the different subconstructs of fractions?* In order to address this, we examined how students deploy subconstructs of fractions in solving problems involving proportions and percentages.

In the following sections, we review the research on how adults understand fractions in the context of proportional reasoning. Then we describe our methods and present the setting of our study followed by the results. In the Discussion and Conclusion, we sum up the key findings and discuss their theoretical and practical implications in the general framework of adults learning mathematics.

**Literature review**

There are many studies that indicate the limited understanding of students of rational numbers and fractions (e.g., Charalambous & Pitta-Pantazi, 2007, Vamvakoussi, Van Dooren, & Verschaffel, 2012). Concerning adults’ understanding of fractions and rational numbers the relevant studies focus on preservice (e.g., Livy & Vale, 2011) or in-service teachers (e.g., Depaepe et al., 2015), and their results refer to a poor understanding of ratio and proportional reasoning among these students. More relevant to our research are: the studies of Baker et al. (2012) and Doyle et al. (2016), set in a community college, the study of Alatorre and Figueras (2005), in which the participants are adults of different ages and schoolings and Ahl’s (2019) study, which refers to adults in a prison education program.

Baker et al. (2012) investigated adults’ competency in fractions in a community college in United States. Their objective was to extend their earlier study (Baker et al., 2009) and to describe the way the understanding of different subconstructs of fractions work together and complement each another in explaining students’ competency. For this purpose, the researchers used quantitative methods, and added a transcript of a small group session.

A basic notion that emerges in a significant number of studies is the notion of a fraction subconstruct that was introduced in Kieren (1980); see the next section. Doyle et al. (2016), in the setting of a community college in United States, present a teaching experiment, in which Kieren’s (1980) fraction subconstructs were used as a basis during their analysis of problem solving. The researchers used quantitative methods, such as a multiple linear regression model, in order to predict or explain students’ competency in formal problem solving. They also used qualitative methods, such as the analysis of transcripts from classroom lectures during the teaching experiment, in order to describe how students used these rational number subconstructs during problem solving with ratio, quotient, proportion, and percent. Their results confirmed that the subconstruct of *part-whole* (see next section) is significantly easier for the adult students than the other subconstructs. Next in difficulty are the *equivalence* and *ratio* subconstructs, and then *operator* and *quotient*. The subconstruct of *measure* follows, and finally problem solving or proportional reasoning is the most difficult of all. As far as it concerns the students’ reasoning, the analysis showed that when the problems included a visual form of the concepts of part-whole such as circles and number lines, measure acted as an accelerator for the students’ reasoning, since the students reasoned correctly because they were able to relate problem information to these pictures. These picture-concepts also underpinned informal reasoning as students expressed strategies and applied processes based upon how the information in the problem was represented. However, see the limitations of this approach in one example in the Results section (e.g., one student’s reasoning about Task 11).

Alatorre and Figueras (2005) describe the answers that adults without primary schooling gave to different ratio and rate comparison tasks. The research took place in a centre for adult education in
The participants were six quasi-illiterate adults, who were interviewed, and the sessions were videotaped. During the interviews, the participants had to answer several questions about problems that demanded proportional reasoning. The problems that included ratio comparison tasks were “easier” for the adults when these included non-proportionality situations (equivalent fractions) than those that included non-proportionality ones (non-equivalent fractions). The results of this study have confirmed that the ability for proportional reasoning develops in the form described by the developmental model that these researchers propose.

Ahl (2019) presents an analysis of a test which was constructed for the assessment of adults’ prior knowledge of proportional reasoning. The participants were adult students of a prison education program in Sweden. The test was administered to 32 adults, and follow-up interviews are provided from three participants. The results indicate that the test could serve as an instrument for the detection of adults’ prior knowledge on proportional reasoning. In addition, the results of this research give information about the ways that adults express notions of proportional reasoning in different contexts.

**Context and methodology**

The setting of our study was a Second Chance School (SCS) in a city of north-western Greece. The Second Chance Schools offer education to adults over 18 years old, who have dropped out of ordinary compulsory education (Vekris & Chontolidou, 2003) and operate as a pathway for them to complete their basic secondary education (Papaioannou & Gravani, 2018). The students of SCSs generally have a low socio-economic background (Lamb et al., 2011), since among them are immigrants, refugees, people from minorities, people with health problems and people who live in poverty. These students come from settings that, in the context of another European country (Germany), are called “vulnerable” (Heilmann, 2019).

The duration of studies in these schools is two academic years and mathematics is taught for three hours per week. The mathematics course for adults, correspond to the first two years of Greek middle school (gymnasium). According to the mathematics curriculum for the SCSs (Lemonidis & Maravelakis, 2013), the structure and the content of the course include proportional reasoning, since among its outcomes we read:

- The understanding of the importance of natural numbers, fractions, decimals and percentages and their use in real problem situations.
- The conversion of one type of rational number to another, for example, the conversion of a fraction to a decimal number and vice versa.
- The development of different strategies for the use of rational numbers in real problem situation.

Concerning fractions, by the end of the course adults are expected to be able to:

- read, write and order simple fractions;
- represent fractions with sets of objects, area modes and number lines;
- convert equivalent fractions, e.g. from 2/4 to 1/2;
- compare fractions with the unit;
- add and subtract fractions with the same denominator;
- convert improper fractions to mixed numbers;
- add and subtract dissimilar fractions (fractions with different denominators) and mixed numbers;
- multiply and divide fractions and mixed numbers;
- connect the notion of the ratio with the fraction.

Our study involved 30 adult students attending the second year of the SCE, 16 men and 14 women, ranging from 25 to 65 years. Most of them were unemployed or unskilled workers. Their
mathematical skills corresponded to a lower secondary level, since they had completed the first year of the SCS. Nonetheless, all were highly motivated to learn (McGregor & Mills, 2012).

For our study we designed a task sheet with 13 open-ended tasks (see the Appendix) and asked the participants to complete it in 1½ hours. The tasks were adapted either from studies in the research field on proportional reasoning or constructed by us. The latter items (three out of the 13) were based on real problems that our students referred to during the teaching of the relevant notions. During the preceding three to four weeks the students had been taught fractions and their different representations, the equivalence of fractions, percentages, addition of fractions with the same denominator and problems that required proportional reasoning.

The tasks were chosen and categorised according to the fraction subconstructs definitions of Charalambous and Pitta-Pantazi (2007) as implemented for adults by Doyle et al. (2016). Charalambous and Pitta-Pantazi (2007) based their study on the assumption that fractions constitute a multifaceted notion encompassing five interrelated subconstructs – part-whole, ratio, operator, quotient, and measure – as proposed first by Kieren (1980) and expanded by Behr et al. (1983) to a theoretical model which linked the fraction subconstructs to the operations of fractions, fraction equivalence, and problem solving. The objective of Charalambous and Pitta-Pantazi (2007) was to examine how students at the two last grades of elementary school understand different subconstructs of fractions. Doyle et al. (2016) used the fraction subconstructs for their study with adult students.

In particular, the part-whole subconstruct refers to the symbolic notation p/q to represent the partitioning of a whole entity into q equal shares and then taking p out of the q shares, the fraction that will be produced must be smaller than the unit. As Figure 1 shows for Task 6 this subconstruct refers to the partitioning of a unit into three equal pieces and then the comparison of the shaded part with the fraction 2/3; see the list of tasks in the Appendix. The point of this particular task is that the three pieces are not equal, in opposition with the fundamental definition of fractions, according to which a continuous quantity or a set of discrete objects are partitioned into parts of equal size (Lamon, 1999).

Does the shaded part of this rectangle correspond to the fraction 2/3? Justify your answer.

![Figure 1: Task 6 (Charalambous & Pitta-Pantazi, 2007)](image)

The ratio subconstruct p/q involves a comparison between the two quantities p and q of the numerator and denominator, respectively. Since it includes the notion of relative magnitude, it is not considered as a number but more as an index of comparison (Behr et al., 1983).

For Task 1 students had to compare lemonade mixtures incorporating in their answers the proportions of sugar and water:

John and Maria are making lemonade. Given the following recipes whose lemonade is going to be sweeter? Justify your answer.

John uses 2 spoons of sugar for every 5 glasses of lemonade. Maria uses 1 spoon of sugar for every 7 glasses of lemonade.

John uses 2 spoons of sugar for every 5 glasses of lemonade. Maria uses 4 spoons of sugar for every 8 glasses of lemonade. (Doyle et al., 2016)

According to Charalambous and Pitta-Pantazi (2007), for this subconstruct the students have to realize that there is a relationship between two quantities and that when these quantities change together (via the multiplication or the division by the same number) the relationship remains constant. In this case, the understanding of equivalent fractions refers to the ratio subconstruct.
The operator subconstruct refers to the process of taking the fraction $p/q$ of some quantity by the multiplication of $p$ with this quantity and then the division of the product by $q$. In Task 2, which is based on a real situation that one of our adult students confronted, the students had to apply the given fraction to a certain number.

Johanna’s child, whose weight is 12 kilograms, is sick and needs an antibiotic. The recommended dose in ml is equivalent to $1/4$ of its weight [in kilos]. How many ml of the antibiotic should Johanna give to her child? Justify your answer.

This subconstruct could be conceived as the result of two multiplicative operations since the numerator is multiplied by a given quantity, followed by the division of the result with the denominator, or vice-versa (Charalambous & Pitta-Pantazi, 2007).

The quotient subconstruct $p/q$ is perceived as the amount obtained when $p$ quantities are divided into $q$ equal shares. In Task 3 the students had to connect a fraction with a division:

Three pizzas are shared equally among four students. What fraction of a pizza will each student receive? (Charalambous & Pitta-Pantazi, 2007).

In this subconstruct there are not any limitations like with the part-whole subconstruct since the numerator can be smaller, equal to or bigger than the denominator and the resulting quantity can be less than, equal to or more than the unit (Charalambous & Pitta-Pantazi, 2007).

The measure subconstruct is connected with the placement of a fraction on the number line. For Task 11 the students had to locate three different fractions on a number line and to justify their answer.

Locate the following numbers on the number line and justify your answer.

![Number Line]

*Figure 2: Task 11 (Charalambous & Pitta-Pantazi, 2007)*

In this subconstruct the fraction has a duality since it corresponds to a number and it is also associated with the measure allocated to some interval (Charalambous & Pitta-Pantazi, 2007). This subconstruct is difficult for the students to understand, since they have to make a qualitative leap when they move from whole to fractional numbers (Lamon, 1999). This observation is affirmed also by other researchers (Vamvakoussi et al., 2012), who connect students’ difficulties with rational numbers to their differences with natural numbers and the disparity between the students’ deeply rooted prior knowledge about the set of natural numbers.

Besides the tasks that referred to the subconstructs of fractions, we used tasks that referred to formal proportional reasoning (Doyle et al., 2016). As reported by these researchers, in a proportion problem someone first has to find the unit rate between the given referents and then to follow a multiplicative strategy; the level at which this procedure can be accomplished constitutes formal proportional reasoning. For example, in Task 7 the student has to calculate the rate of the medicine and the water and then to convert it in an equivalent fraction in order to answer the question:

If 0.5 ml of medicine is mixed with 2 ml of water to form a solution, what is the ratio of drug to water in the simplest terms? Justify your answer. (Doyle et al., 2016)

Although a mathematical task can serve as a window into the students’ thinking (Hunting, 1997), it cannot give enough information about the cognitive processes the student followed in order to answer the task (Ginsburg, 2009). For this reason, we have decided to deploy a mixed methods approach (Sammons & Davis, 2016). In particular, we conducted “clinical interviews” with five of the participants, three women and two men, in order to have more data about their ways of thinking. The
interviews were semi-structured, lasted for approximately 40 to 50 minutes and were audio taped. During the interviews we asked the participants to reason aloud about their answers to the tasks. By that method we had the opportunity to enrich our data since the participant had the opportunity to explain why s/he answered in a particular way or even to amend her/his answer to try to find the correct one.

Results

As we stated earlier, the main objective of our study was to identify the processes that adult students in an SCS use when confronted with mathematical tasks related to fraction subconstructs and to formal proportional reasoning, and to monitor the difficulties that these students experience. In general, the tasks that referred to the subconstructs of ratio (T1, T5) and part-whole (T6) were the easiest, in the sense that the majority of the participants answered them correctly; see Table 1. The most difficult tasks were those that referred to the measure subconstruct (T11, T12) or involved formal proportional reasoning (T8, T13). Table 1 below summarises the results in all tasks. The first column refers to the fraction subconstructs in general and to the formal proportional reasoning. The second column indicates the corresponding items. The strategies used by the students are outlined in the third column while the fourth column includes the number of answers correct and justified (rather than merely stated without justification).

We now present a representative response to a task for each subconstruct with relevant transcripts from the interviews.

As we stated above, the task which elicited the best performance was T6, which referred to the part-whole subconstruct. Below Helen (all names are pseudonyms) explains why she answered negatively:

Helen: No, because the middle one is smaller than the other two parts.
Teacher: So, what?
Helen: I don’t think that this represents the 2/3.
Teacher: If you had to draw it how would you do it?
Helen: I would make a similar one, but I would make three equal parts. I would divide it to three equal parts, and I would take the two of them (as she speaks, she draws the correct shape).

Helen had understood that a fraction describes a situation, in this case a rectangle, in which a quantity or a unit is divided into equal parts. She divided the rectangular correctly and she stated that the parts must be equal.

Task 1 referred to the ratio subconstruct. For this task most of the students computed fractions, used the notion of ratio, and then tried to compare them. In order to make the comparison, some of them converted the initial fractions, while others based their answers on the relationship of the numbers that constituted the initial fractions. Maria below explains how she converted the initial fractions, in part 1 of the task, in order to compare them:

Maria: I get the largest denominator and I put it in the first fraction and the second denominator in the second fraction. I do it in reverse. And then I multiply 2×7=14 and 5×7=35. I continue the next and compare 14/35 with 5/35. Surely 14/35 is larger, since it has the bigger numerator. The sweeter lemonade is John’s.

In her answer it is obvious that she tries to compare the initial ratios by converting to equivalent fractions. After this conversion, she was able to compare them and answer the question. (We may say that she compared the ordered pairs using multiplicative rules.)
Table 1: Strategies used by the students

<table>
<thead>
<tr>
<th>Fraction subconstructs</th>
<th>Items</th>
<th>Strategies used by the students</th>
<th>Number of justified answers (Total N=30)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part-whole</td>
<td>T6</td>
<td>Detect that the bar is not divided to equal parts.</td>
<td>29</td>
</tr>
<tr>
<td>Ratio</td>
<td>T1</td>
<td>Compute fractions and try to compare them by converting them to equivalent fractions.</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Divide the numerator with the denominator.</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Convert the fraction to a decimal number.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Find relationships with the numbers on the numerator and the denominator. Compare the fractions with a unit fraction.</td>
<td>3</td>
</tr>
<tr>
<td>Ratio-equivalence</td>
<td>T5</td>
<td>Convert the fractions to equivalent ones.</td>
<td>20</td>
</tr>
<tr>
<td>Operator</td>
<td>T2</td>
<td>Apply the fraction to the given number</td>
<td>20</td>
</tr>
<tr>
<td>Quotient</td>
<td>T3</td>
<td>Compute the fractions and compare them.</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>T4</td>
<td>Compute the fraction and divide the numerator with the denominator.</td>
<td>10</td>
</tr>
<tr>
<td>Measure</td>
<td>T11</td>
<td>Divide correctly the number line and locate the fractions on it.</td>
<td>3</td>
</tr>
<tr>
<td>Measure</td>
<td>T12</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Formal proportional reasoning</td>
<td>T7</td>
<td>Apply rational number concepts to problems that include proportions and percents.</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>T8</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>T9</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>T10</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>T13</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>
A few students divided the numerator with the denominator interpreting the fractions as quotients. Others tried to find how many spoons of sugar are used for every glass of lemonade by using proportions and trying to compare magnitudes. In the next transcript, Yanna explains her way of thinking for part 2 of the task. She did not use fractions but instead she converted to proportions and tried to compare the magnitudes by using multiplicative thinking.

Yanna: It is 2 spoons of sugar in 5 glasses of lemonade and 4 spoons of sugar for every 8 glasses of lemonade, so the 8 glasses have 4 tablespoons of sugar and the 5 glasses… Because in the 8 glasses it is about 1 spoon in every 2 glasses while here at 5 comes less than half of a spoon for every 1 glass of lemonade. Isn’t it?

Whilst the initial ratio was equal to 2/5, Yanna tried to convert it to the equivalent unit fraction, which would be easier for her to make the comparison.

For Task 2, which referred to the operator sub-construct, the students had to apply a fraction to a given quantity. Most of the students managed to calculate the correct result by multiplying the fraction with the given number, as we can see in Figure 3. The student’s written answer reads: “one of the four is equal with three because we divide the 12 kilograms and we take one of these” (translated from the Greek).

Some of the students divided the given number with the denominator and found the result.

Helen: 3 yes, the 1/4 is the 3.
Teacher: How did you think it? Why is the 1/4 equal to the 3?
Helen: Because 3 times 4 is 12. I split the 12 to four pieces. So, she will have 3 ml.
Teacher: Why did you decide to divide it to four pieces?
Helen: Because it says that she wants the 1/4 of the child’s weight.

In Helen’s answer, it is evident that she interpreted the fraction as the partition of the given number to four equal units.

Task 3 referred to the quotient sub-construct:

Three pizzas are shared equally among four students. What fraction of a pizza will each student receive?

This was an easy task since all students were able to answer it correctly, although only 15 justified their answer (see Table 1). Many of them designed three rectangles representing the three pizzas (Figure 4) and divided them to four pieces. These students used the visual representations of the fractions. In Figure 4 a student writes “we cut the pies in four pieces and every child takes three pieces”, although he makes a formal mistake since he writes that 3/4 is equal to 4/3.
The most difficult tasks (T11, T12 see Appendix) for the students were those that referred to the measure subconstruct, since only three of the students answered them correctly. These students based their answers on the comparison of the fractions and the division of the number line. For T11, all of them used additive reasoning since they wrote that three parts, or six parts, make the whole but only one used multiplicative thinking since he highlighted the fact that 1/3 is the double of 1/6.

Jim: Here we have a ruler… We counted, and we found that for the fraction 1/6 we cut the ruler to 6 pieces and we took one of the six. Here the four, the five of the six and the four of the three. Isn’t that so? I have divided one piece into three pieces, and I moved further, I said, plus 1. One more than the whole unit, namely 4 out of three.

Teacher: For the fraction 4/3 you said one more than?
Jim: One more than the unit, one more than the 3/3.
Teacher: So, you understood every line once equal with the 1/6 and once with the 1/3.
Jim: Yes.
Teacher: Why did you place the 5/6 there?
Jim: Because I divided this piece to 6 equal parts, and I chose the five of the six.

In Jim’s answer it is obvious that he used additive reasoning since he added the 1/6 in order to construct the fraction 5/6. He had understood what a fraction constitutes but it was difficult for him to explain the relation between the 1/3 and the 1/6. However, he understood that a fraction bigger than the unit has a bigger numerator than denominator and he understood how to convert the unit to a fraction with equal numerator and denominator.

As we mentioned before only one student used multiplicative thinking in his answer to T11. Below we present a transcript of the interview with him.

Teacher: How did you think it through?
George: Here is the 0 and we move right and left. It gives us the 1, the 1/3 and we are asked to find the 1/6.
Teacher: Why did you place there the 1/6?
George: We start and we count from the 0…. The 1/3 is here…. When we move to the left is minus and when we move to the right is the plus. The 1/6 if we start and count 1, 2, 3, 4, 5, 6 …
Teacher: Why did you write that “the 1/3 is the double of 1/6”?
George: Because if we have 6… the 1/3 is the double of 1/6. We want to find the 1/6, we have the 0 and we search for the … In order to find the 1/3 is two times the 1/6.

George in his answer used multiplicative thinking since he realised that 1/3 is the double of 1/6. In order to explain his answer, he drew six lines to denote the 1/6 and three for the 1/3. He was able to assign the 1/3 to the correct interval and to detect the relationship between the 1/3 and the 1/6 on the number line. So, he came to the right conclusion using relational thinking. In the next transcript George compares correctly the 4/3 with the 1 using additive thinking.

Teacher: Why did you place the 4/3 after the 1?
George: Because we have 1, 2, 3 and 4 thirds. I add the 1/3 until I reach the 4/3. Every piece corresponds to 1/3.

Besides the tasks that referred to fraction subconstructs, we used tasks that included formal proportional reasoning. Some of these were relatively easy for the students (T7, T9 and T10) while others were rather difficult (T8, T13); see Table 1. Most students answered correctly Task 10, maybe because it described a situation they confronted in their everyday life. They multiplied the percent with the given number, or they used the equivalent fractions (Figure 5):

In the supermarket Gouda cheese has 48% fat, how much fat will we get if we consume 300 grams of it? Justify your answer.

Figure 5: Task 10 sample solution

Maria: The 100 grams have 48, we want to find about the 300 so we will multiply the 48 with the 3 which is equal with the 144.
Teacher: Why did you say that in the 100 grams we have 48? How did you get this?
Maria: Because it says 48%.
Teacher: Why did you multiply with the 3?
Maria: It is three times the 100.
Teacher: Did you use as a base a certain number?
Maria: Yes, the 100.

In Maria’s answer it is obvious that she had understood the method for calculating the percent from a given number. Also, she had understood that the base-whole of the percent is the 100 which she used as a reference point for the calculation.

Task 8 was a bit difficult for the students.

Task 8 (formal proportional reasoning): Which of the following fractions is the closest to 1?
   a) 2/3   b) 3/4   c) 4/5   d) 5/6

Most of them answered that 5/6 is the closest to 1, but they did not correctly justify their answer. In the next transcript, Yanna tries to explain her thinking. We may say that she understands the fraction intuitively but cannot express it with mathematical notions. For this reason, she was classified as producing a “justified answer” in Table 1.

Yanna: 5/6 is closer to 1, then 4/5, then 3/4 and then 2/3.
Teacher: The answer you gave is correct, but how did you think about it?
Yanna: Because we make a pie and divide it into 6 pieces if I eat the 5 I get the most. At 4/5 you will get less. The other one who gets the 3/4 will get less and the other who gets the 2/3 he takes even less. So, when I get the most, I’m closer.
Teacher: Closer, to where?
Yanna: To 1.
Teacher: What does 1 represent for you?
Yanna: 1 is the whole.
Few students justified their answer by using either the equivalence of fractions for their comparison, or the subconstruct of quotient. One student represented visually the fractions and based her answer on the fact that the piece left from the 5/6 seemed to her to be the smallest one. Figure 6 presents her answer, where she states that “the closest is the 5/6”.

![Figure 6: Task 8 sample solution](image)

However, the reader will notice that relying on these diagrams is not convincing, since the relevant parts of the last circle, for example, are not equal. We could ascribe this fact to the difficulty that these students had to express their thoughts with the mathematical language.

**Discussion**

Our study focused on proportional reasoning during problem solving in the setting of a Second Chance School. The participants of our study were adult students who had been away from school mathematics for many years and returned in order to complete their secondary education. Since proportional reasoning is inextricably connected with the rational numbers, we sought answers to a research question about the range of students’ understandings of the different subconstructs of fractions. In order to achieve this, we examined how students deploy subconstructs of fractions in solving problems involving proportions and percentages.

Although at the beginning of the lessons the adult students expressed their anxiety and fear concerning the notion of fractions, at the end they seemed able to incorporate an understanding of the properties of fractions in order to reason, at least to some extent, proportionally. We could claim that our students overcame their previous experience of mathematics which is a significant goal of learning mathematics as an adult (Wedege & Evans, 2006). Concerning their performance, most of them easily solved the tasks that required the formulation of a fraction either from a sentence or from a pictorial representation. They were also able to construct ratios based on the data of the task and to convert them to equivalent ones. The tasks that corresponded to the subconstructs of ratio, operator, quotient, and part-whole were relatively easy for most of the students since they managed to establish relationships between quantities, and then those relationships (ratios) became the new elements with which they operated.

At the same time, most of them had difficulty with the tasks that referred to the measure subconstruct, since these tasks require both an understanding of the splitting and iteration processes, a finding that is similar to that often found among younger students (Charalambous & Pitta-Pantazi,
2007). Very few students answered these and in their answers they used additive thinking instead of multiplicative reasoning.

The students also found difficult some tasks that required formal proportional reasoning. In particular, this observation refers to tasks which included the finding of a ratio through its implementation in a given quantity or the placement of unlike fractions in ascending order. At the same time, the students’ performance was very good in tasks related to situations of their everyday life. Since proportional reasoning is an ability which is known to take a long time to develop, we may claim that the participants’ performance was relatively satisfactory. According to Boyer and Levine (2012), the necessary skills for proportional reasoning are multiplicative and relational thinking, supplemented by a highly developed understanding of foundational concepts such as fractions, decimals, multiplication, and division. Obviously, the students in our study had not developed entirely the above skills; that is why they did not rely solely on their mathematical knowledge, but they also drew on their out-of-school experience when they returned to school (Evans, 2000).

Conclusion

Proportional reasoning has such a critical role in a student’s development of mathematical thinking that it has been characterized as a cornerstone of higher mathematics and the capstone of elementary concepts (Lesh, Post, & Behr, 1988). Additionally, it has been characterised as a “pervasive activity that transcends topical barriers in adult life” (Ahl et al., 1992, p. 81), therefore there is no doubt that it should be included in the mathematics courses that are taught to adults who return to school. The results of our study have shown that realistic contexts are quite helpful, especially because they assist adults to overcome their anxiety by relating the given situation to their everyday lives. Thus, adult educators should have this in mind, at the same time considering other factors, such as the participants’ backgrounds (including socioeconomic factors), in order to design relevant and interesting contexts. From a mathematical point of view, we have seen these adults’ difficulties with particular subconstructs, such as the measure subconstruct and with mathematical processes such as formal proportional reasoning. These difficulties are probably related to their prior learning experiences.

Summing up, our study has shown that achieving a balance between applicability to everyday life and mathematical richness should be a main goal for mathematics educators at all levels, including adult education.

References


Kontogiannni & Tatsis Proportional Reasoning of Adult Students in a Second Chance School: The Subconstructs of Fractions.


McGregor, G., & Mills, M. (2012). Alternative education sites and marginalized young people: ‘I wish there were more schools like this one’. International Journal of Inclusive Education, 16(8), 843-862.


Appendix: Task Sheet

**Ratio**

Task 1 (Doyle et al., 2016)

John and Maria are making lemonade. Given the following recipes whose lemonade is going to be sweeter? Justify your answer.

1) John uses 2 spoons of sugar for every 5 glasses of lemonade. Maria uses 1 spoon of sugar for every 7 glasses of lemonade.

2) John uses 2 spoons of sugar for every 5 glasses of lemonade. Maria uses 4 spoons of sugar for every 8 glasses of lemonade.

**Operator**

Task 2 (constructed by the authors)

Johanna’s child, whose weight is 12 kilograms, is sick and needs an antibiotic. The recommended dose in ml is equivalent to 1/4 of its weight. How many ml of the antibiotic should Johanna give to her child? Justify your answer.

**Quotient**

Task 3 (Charalambous & Pitta-Pantazi, 2007)

Three pizzas are shared equally among four students. What fraction of a pizza will each student receive?

Task 4 (Charalambous & Pitta-Pantazi, 2007)

If 3 pizzas are shared evenly among seven girls, while 1 pizza is shared evenly among three boys, who gets more pizza, a girl or boy?

**Ratio - Equivalence**

Task 5 (Charalambous & Pitta-Pantazi, 2007)

Can you fill in the gaps and justify your answer?

\[
\begin{align*}
\frac{2}{3} &= \_ ; \\
\frac{25}{40} &= \frac{5}{9} \quad ; \\
\frac{7}{9} &= \frac{42}{\_} ; \\
\end{align*}
\]

**Part-whole**

Task 6 (Charalambous & Pitta-Pantazi, 2007)

Does the shaded part of this rectangle correspond to the fraction 2/3? Justify your answer.
Measure

Task 11 (Charalambous & Pitta-Pantazi, 2007)
Locate the following numbers on the number line and justify your answer.

\[
\begin{array}{ccc}
\#1 \frac{1}{6} & \#2 \frac{4}{3} & \#3 \frac{5}{6} \\
0 & \frac{2}{3} & 1
\end{array}
\]

Task 12 (Charalambous & Pitta-Pantazi, 2007)
Locate the 1 in the number line and justify your answer.

\[
\begin{array}{c}
0 \quad \frac{5}{9}
\end{array}
\]

Formal proportional reasoning

Task 7 (Doyle et al., 2016)
If 0.5 ml of medicine are mixed with 2 ml of water to form a solution, what is the ratio of drug to water in the simplest terms? Justify your answer.

Task 8 (Doyle et al., 2016)
Which of the following fractions is the closest to 1?

- a) 2/3
- b) 3/4
- c) 4/5
- d) 5/6

Task 9 (constructed by the authors)
Mary used exactly 15 paint tins to paint 18 chairs. How many chairs she could paint with 25 cans? Justify your answer.

Task 10 (constructed by the authors)
In the supermarket Gouda cheese has 48% fat, how much fat will we get if we consume 300 grams of it? Justify your answer.

Task 13 (constructed by the authors)
Please fill in the gaps and justify your answer.

The … (a fraction) of 200 equals 150.

The … (a fraction) of 200 equals to … (a number larger than 100).

The … (a fraction) of 200 equals to … (a number smaller than 100).
Safety First: Combining Task Models of Medical Devices with Numeracy Skills and Technical Competence

Judy Bowen
University of Waikato, New Zealand
<jbowen@waikato.ac.nz>

Diana Coben
King's College London and University of East Anglia, UK
<diana.coben@kcl.ac.uk>

Abstract
We propose that by more closely aligning interdisciplinary work in (a) numeracy education for medication dosage calculations and (b) model-driven design for medical devices that are used for delivery of medication we may help address the incident-rate in incorrect medication calculations and delivery, given that such devices commonly require the user to engage with numerical information via a digital interface. We demonstrate the use of task models as a way of supporting safe, effective and efficient delivery of medication to the patient, taking as our example the use of infusion and syringe pumps in Nursing. This work indicates a new way of facilitating knowledge transfer between numeracy education and medical device design and usage, using task models. We aim to support medical professionals’ and students’ numeracy education as well as to inform the design of medical devices based on a better understanding of the use and potential errors of medication delivery by trained professionals.

Key words: Task models; safety-critical systems; Nursing numeracy; syringe pumps; human-centred computing; interaction design.

Introduction
In the course of their work, Nurses and other healthcare professionals frequently engage with medical equipment via digital interfaces. Those interfaces typically involve activities such as entering numbers, checking that equipment is correctly calibrated, measuring and recording data, etc., and sometimes calculation. Patient safety is paramount so the stakes are high.

This article outlines interdisciplinary work in progress bringing together the authors’ research interests and expertise in computer science and adult numeracy in safety-critical work contexts in order to address an important issue: how best to support safe, effective and efficient delivery of medication to the patient when that delivery involves the healthcare professional engaging with medical equipment via a digital interface?

Our research interests and expertise are as follows. Bowen is a Computer Science researcher in formal methods and modelling for safety critical software. She has been focusing on medical devices such as syringe drivers and infusion pumps for the past seven years. She is interested in users, usability, correctness and the safety properties of medical devices. Coben is a specialist in adult numeracy
practices and education, especially in safety-critical work contexts, who has a particular research interest in numeracy for Nursing spanning 25 years.

In the safety-critical context of Nursing, Bowen’s research has focused on: engineering interactive safety-critical systems such as medical infusion pumps and syringe drivers; seeking to ensure devices behave correctly at all times; and seeking to improve design so that devices are easier and safer to use. Meanwhile, Coben’s research has sought to identify and support the development of numeracy skills required to practise safely, effectively and efficiently, including calculating and delivering medication dosages correctly using diverse delivery mechanisms, many of which have digital interfaces. She is a member of an international interdisciplinary team investigating medication dosage calculation problem-solving (MDC-PS).

We bring together concepts of technology design (both functional correctness and usability concerns), numeracy and medication delivery competency. We consider the use of task models (outlined below) as a step towards achieving this. Our wider research seeks to address the following questions:

• How can we use formal models of medical devices in conjunction with task models to improve the design and development process?
• How can we use task models to identify mismatches in user knowledge and education with device usage?
• How can we use task models of medication calculations and technical competence to inform user education and the design of medical devices?
• How can we use comparisons of task models of devices and medication delivery to identify potential use errors?

The work in this paper outlines the groundwork needed to answer these questions by examining the use of task models in the domains of medical device design and usage in the context of the development of competency for nurses. These questions relate to all medical personnel. In this paper we focus on nurses because they are most likely to deliver medication directly to the patient: as such, they are the last line of defence in medicines management in the healthcare context (Gottlieb, 2012).

Although we discuss a number of different types of models in this work, the key focus is on task models, which we will use in a number of different ways. Task models are a description of a goal that a user wishes to achieve. They can be used abstractly to describe user actions as a set of steps and subtasks required to achieve the goal (via decomposition of the high level goal) and can also be used to describe concrete steps and actions a user may take to achieve that goal (for example, when using a medical device). Task models provide documentation comprising flexible and expressive notations with precise semantics and support effective design of devices and their evaluation in terms of usability (Paternò, Santoro, & Spano, nd). We believe task models have the potential to be a useful tool for educators seeking to support safe and effective healthcare numeracy practice through authentic education. We give a more detailed description of task modelling later.

**Problems with medical devices**

Common problems with medical devices include:

• Number entry: e.g., confusing methods of number entry; varying methods of number position and decimal point handling; number roll-over problems;
• ‘Same but different’: i.e., devices that look identical but have different firmware, leading to subtle differences in use;
• ‘Similar but different’: i.e., devices that look similar but behave differently.
In this paper we focus on syringe drivers. These are devices designed to deliver intravenous medication to the patient automatically, obviating the need for multiple injections. Intravenous medication forms a major part of hospital inpatient care throughout the world, and most of this is delivered via infusion pumps and syringe drivers. These and other medical devices are considered to be safety-critical interactive systems. That is, they are systems or devices that involve user interaction and which have the potential to be hazardous if they malfunction or if user errors occur. Infusion pumps and syringe drivers have been implicated in numerous adverse events (i.e., injuries "resulting from a medical intervention", Kohn, Corrigan, & Donaldson, 2000), the results of which range from minor patient inconvenience through to serious harm or even death (FDA, 2017).

A recent UK example illustrates the problem of ‘same but different’ devices. In this case, the devices are the Graseby MS16A and MS26 syringe pumps, both of which were used in UK palliative care until they were phased out in 2015. An article in *The Daily Telegraph* newspaper reported:

> While one model is set to deliver opioids over 24 hours, the other, which looks identical, performs the same task in 60 minutes, meaning patients risk being given a day’s worth of drugs in just one hour.
> Doctors have said they resorted to sticking on makeshift aluminium strips so they could tell the difference.
> [...] Dr Richard Ian Reid [...] told police he could not remember which Graseby device was which, saying they were “totally confusing” and “really dangerous”. (Bodkin, 2018)

This is symptomatic of an ongoing global problem. Between 2005 and 2009 the FDA received approximately 56,000 reports of adverse events associated with the use of infusion pumps, including numerous injuries and deaths. In the same period manufacturers recalled 87 infusion pumps to address identified safety concerns. Since 2009 the situation has improved very little, despite increased focus on these issues.

Underlying causes of these problems include: user interfaces that are not fit for purpose; inappropriate interaction between elements of devices; and issues in user engagement with devices - all factors that are not treated by manufacturers or regulators with the same rigour as the underlying functionality of the device. The lack of a common method for verifying device correctness prior to FDA or other regulatory approval also causes problems, as does the absence of common methods for reasoning about user interaction at a formal level to allow for model-checking and theorem-proving.

The FDA has launched a project aimed at improving this situation (the Generic Infusion Pump project\(^1\)) which focusses on the design and development of the technology. This work, and similar research in this domain, is based around either software engineering - seeking to ensure the devices behave correctly at all times (Campos & Harrison, 2011), or usability research - seeking to improve the design so that devices are easier to use (Thimbleby, 2015). However, there are alternative, complementary approaches in other disciplines which have the same goal of reducing medication delivery errors. These include training medical personnel in the use of medical devices (Vipond, 2016) and in the numeracy skills required to calculate medication dosages correctly for different delivery mechanisms (Coben & Weeks, 2014) in order to ameliorate the risk of user error.

When we focus on ‘user error’ as a technological problem we make assumptions. For example, if an error is made leading to the wrong amount (volume) of medication being delivered to a patient we may assume that the medical professional setting up the infusion (whom we henceforth refer to as the ‘user’) has started with the correct values for setting up an infusion and made a number-entry error. Indeed, number-entry errors are a common occurrence in this domain (Thimbleby, 2015). However, in

---

1 https://rtg.cis.upenn.edu/gip/
practice it may be impossible to determine at which point the error has actually been made. It may be that rather than a number entry error, it is in fact a calculation error in which the user starts the infusion set-up with wrongly calculated values that they subsequently enter into the device correctly.

Numeracy education for medical staff focuses on ensuring that they are able to set up and perform the necessary calculations to convert prescribed medication into the appropriate values and measures for their delivery mechanisms, which may be tablets of a given strength (e.g., 5mg), or liquid solutions containing given amounts of medication (e.g., 20mg per 2ml). It is essential that this precursor to actual medication delivery is correct and that the medical personnel can then safely and correctly deliver the correct medication doses using the technology provided.

**A possible solution**

We are working towards finding an effective solution to the problem outlined above through our proposed programme of interdisciplinary research. This has two main elements: engineering interactive systems; and authentic numeracy education based on a sound model of competence. The engineering element entails using model-driven development techniques for interactive systems in the design and implementation of medical devices. Models of software and hardware/software hybrid systems (or cyber-physical systems) are abstract descriptions focussing on specific properties of that system. The models can be at differing levels of formality where informal designs and prototypes can be used to describe the ‘look and feel’ of the interface to a system to non-technical stakeholders; and fully formal mathematically-based models of functionality can be used for formal reasoning based on logical tools such as model-checkers. The authentic numeracy education element is based on a sound model of competence which was developed for medication dosage calculation problem solving (MDC-PS), and based on research set out in the ‘Safety in Numbers’ Virtual Special Issue of *Nurse Education in Practice*². These two elements are described below.

Figure 1, below, gives an overview of our approach. The arrows are labelled with the key steps of our research. Step 1 relates to research questions 1 and 2 above. In this paper we describe how we can use existing models of interactive systems (particularly formal models of safety critical systems) (Bowen & Reeves, 2008, 2013) to derive task models and interaction sequences, and how this provides information about use and user knowledge. We discuss Step 2 from the perspective of technical competence models designed to inform numeracy education for healthcare personnel, particularly nurses (Weeks et al., 2013). Finally, we demonstrate how questions 3 and 4 can be addressed by relating these diverse task models to each other and comparing them (Step 3).

This work indicates a new way of facilitating knowledge transfer between numeracy education and medical device design and usage, using task models. We aim to support healthcare professionals’ and students’ numeracy education as well as to inform the design of medical devices based on a better understanding of the use of, and potential errors in medication delivery by trained healthcare professionals.

We discuss this next in the context of medication dosage calculation problem solving (MDC-PS) in Nursing.

---

² [https://www.nurseeducationinpractice.com/content/safety](https://www.nurseeducationinpractice.com/content/safety)
Weeks et al. (2013) have proposed a competence model for MDC-PS which represents the intersection between the ability to interpret the dosage calculation problem and accurately set up rate equations (conceptual competence), the correct calculation of accurate numerical values for the dose and rate of administration (calculation competence) and the selection of appropriate measurement vehicles and accurate measurement of the dose and rate of administration (technical measurement competence), see Figure 2 (Weeks et al., 2013, p. e25).

Specifically, the MDC-PS competence domains are characterised as follows:

1. Conceptual competence refers to the need to:
   - Understand the elements of prescription charts, dispensed medication labels and medication data sheets and monographs, and to subsequently extract the numerical information necessary to set up the dosage problem correctly.
   - Position the numerical information appropriately and correctly in an equation format for calculation.
2. Calculation competence refers to the need to:
   - Correctly apply arithmetical operations and compute an accurate numerical value within a safe and acceptable tolerance range for the prescribed medication dose, and/or rate of administration.

3. Technical Measurement competence refers to the need to:
   - Select an appropriate medication administration measurement vehicle (tablet or capsule, oral liquid medicine measurement cup, syringe, infusion pump, etc.)
   - Accurately transform the calculated numerical value to the context of the measurement device/formulation and measure the correct dose of prescribed medication; and/or administer the correct rate of prescribed medication/IV infusion fluid.

The competence model for MDC-PS comprises the intersection between: the ability to interpret the dosage calculation problem and accurately set up rate equations (conceptual competence); the correct calculation of accurate numerical values for the dose and rate of administration (calculation competence); and the selection of appropriate measurement vehicles and accurate measurement of the dose and rate of administration (technical measurement competence). An uncorrected error in any one or more of these domains will result in a medication dosage error in the practice setting (Weeks et al., 2013). Technical measurement competence is particularly relevant to our focus in this paper since it involves the use of medication delivery devices.

**Model-based design for interactive systems**

Model-based design and engineering for safety-critical software and hardware (such as the syringe driver) take many forms and can be considered across two different dimensions. The first dimension is the type of model and where it falls on the scale of formality - from fully formal approaches, such as the use of formal specifications, verification, refinement, etc. (Blandford et al., 2011; Bowen & Reeves, 2007; Harrison, Masci, Campos, & Curzon, 2017) to the informal methods of user-centred design and
more ‘agile’ design approaches (Blandford, Buchanan, Curzon, Furniss, & Thimbleby, 2010) as well as everything in between (Calvary, Coutaz, & Thevenin, 2001). The second dimension is how, and where, the models are used: at the design stage before any implementation occurs; as part of the final testing and sign-off of the implemented solution; or throughout the design process.

Other research approaches seek to consider the ranges of both dimensions by focussing on the integration of formal specification techniques (ideally suited in safety-critical domains) with user-centred design approaches (ideally suited for interactive systems) (Bowen & Reeves, 2006). This allows consideration of both design and functionality using formal and informal methods at the requirements stage and throughout development. It also lends itself to integrated testing approaches as well as post-implementation and reverse-engineering analysis methods (Bowen, 2015; Bowen & Reeves, 2008). While much of this work does not explicitly incorporate task models, they are usually considered as implicit inputs into the user-centred design approach that forms the basis for user interface (UI) and interaction models.

**Task analysis and task models in interactive system design**

Task analysis is aimed at understanding how a user completes a defined task. It allows us to both analyse what a person is required to do to achieve a certain goal (the task) as well as analyse the effort (both cognitive and physical) required to do this. There are a large number of methods, notations and tools used for task analysis within both computer science and psychology (where the origins of task analysis can be found in applied behaviour analysis). The choice of which to use typically depends on the formality of the design process and the use of the task model within that process.

While basic hierarchical decomposition may be sufficient in the early stages of development to support understanding of user requirements, and proved popular when task analysis began to be incorporated into the domain of human-computer interaction (HCI) (see Shepherd, 1989, for example), the use of task models has evolved within interactive system design and development to include goal-based analysis and conceptual models which are used at various stages throughout the design life-cycle. Work by Paternò et al. has extended this further to a comprehensive notation for both task and dialog modelling based on concurrent task trees (CTT) (Paternò, Mancini, & Meniconi, 1997) which includes a variety of logical and temporal operators for ordering and iteration, as well as tools to support the creation of, and reasoning about, such models (Mori, Paternò, & Santoro, 2002).

Palanque and colleagues have incorporated these CTT into a petri-net based modelling and development environment for use within safety-critical interactive system design (Barboni, Ladry, Navarre, Palanque, & Winckler, 2010; Martinie, Palanque, Ragosta, & Fahssi, 2013). These types of extensions, and others (e.g., Dittmar & Forbrig, 2003) allow the use of straightforward hierarchical models for complex reasoning about safety, user collaborations, reconfigurable human-machine interfaces, etc.. As such, their work demonstrates the flexibility of task modelling and the ability to incorporate it into formal modelling for a wider range of uses. This supports the goals of our work and motivates us in the use of task models in the manner we propose.

Of most relevance here is the use of Palanque and colleagues’ petri-net environment to support training of operators of safety-critical systems. This takes a similar approach to our own, by using task models to link specific device models to another domain (in their case, training procedures and programs) (Martinie, Palanque, Navarre, Winckler, & Poupart, 2011). While their aims are different in that they seek to develop appropriate training programs by integrating task models within the simulation environment PetShop (Palanque, Ladry, Navarre, & Barboni, 2009) the use of task-models to bridge cross-domain knowledge demonstrates their applicability in such approaches.
Integrated tools and simulation environments such as PetShop provide the ability to incorporate different concepts and approaches (in this case, task models, interactive system models and training procedures) into a single environment. However, the downside of this is that in order to take advantage of such a tool, everything has to be modelled and developed within this one tool. Frequently, the diversity of artefacts within interactive system development and the heterogeneity of different groups within the design team means this is not the most suitable choice.

Unlike the approaches seen in Instructional Task Analysis, which use task decomposition as a means to decide what skills and knowledge are required by users of a particular system, here we look at the skills and knowledge of clinicians required to administer medication and compare them with the tasks of using technological systems to deliver such medication.

Modelling users

Although task models describe user behaviours, these are at a level of the actions required to achieve a goal. That is, they typically assume correct or optimal behaviours. Comparisons between such models and actual user behaviours in practice can be used to identify, and even model, errors based on missteps or slips (see Johnson, 2011, for example) but task models alone do not identify such errors. More frequently they might be used as part of approaches such as “key-stroke level models” (KLM) (Card, Moran, & Newell, 1980) and “goals, operators, methods and selection” (GOMS) (Card, Moran, & Newell, 1983) which seek to identify cognitive load and effort, which in turn may suggest potential for error.

In contrast, work that does seek to model user cognition and identify potential errors based on this makes different types of assumptions. Blandford et al. have focussed on the effect cognition has on user behaviour (Blandford, Butterworth, & Good, 1997) and extended this to consider distributed cognition for use with multi-user systems (Blandford & Furniss, 2005). More recently they have looked at the effect distributed cognition has on healthcare practices (Berndt, Furniss, & Blandford, 2015; Rajkomar & Blandford, 2012) both in clinical settings and in the home. We also consider the work of Curzon et al. who use salience as an important property in understanding cognition and the effect this has on interactive system design (Rukšėnas, Back, Curzon, & Blandford, 2008). These examples categorise different types of causal effects (distributed behaviours, salience) and their potential to lead to error and then use these to either improve processes or improve the design of safety-critical systems. Rather than consider user behaviours based on such concepts, we instead focus on the driving factor, the key knowledge that users have (or should have) prior to undertaking particular tasks and the competence they demonstrate in performing these tasks. This professional knowledge base has been synthesised in the European Qualifications Framework for Lifelong Learning (EQF) as “The proven ability to use knowledge, skills and personal, social and/or methodological abilities, in work or study situations and in professional and personal development” (European Communities Education and Culture, 2008). This notion of professional clinical competence underpins our research.

Creating links using task models

Formal models of medical devices

As discussed in the previous section, a variety of different models and modelling techniques exist for interactive systems. Here we focus on the presentation model approach described in Bowen and Reeves (2017) which uses several different models at varying levels of abstraction throughout the design and development process. This enables both formal verification of properties such as safety, as well as supporting prototyping and lightweight UI design. Interface designs can be described by the interactive elements (widgets) of the design and their intended behaviours. As such, a formal structure can be given to the narrative behind prototypes, personas, storyboards, etc., which (among other things) removes
ambiguity. Although a full explanation of these is beyond the scope of this paper we introduce the basic elements by way of an example which we subsequently expand upon.

Consider the syringe driver shown in Figure 3. This is a modal device, that is, the behaviour of each of its buttons is dependent on the current mode of the system. A simple presentation model can be constructed which describes each mode as a collection of the available widgets. These are described in a tuple (a finite sequence of elements) giving a name to the widgets, their type (input/output) and their intended behaviour. For example, the device has a mode which enables a user to enter the volume of medication to be infused, which we call ‘SetVolume’. The presentation model of the ‘SetVolume’ mode might include:

```
SetVolume is
OnButton, ActionControl, (I_Init),
UpButton, ActionControl, (S_IncVolume),
DownButton, ActionControl, (S_DecVolume),
Display, Responder, (S_IncVolume, S_DecVolume),
```

as well as the rest of the widgets, which we omit here for brevity. Each mode of the device is described in a similar way. The button behaviour names are prefixed with either an “I_” which indicates it is a behaviour relating to interface navigation (mode change) or an “S_” if it relates to system functionality.

The lightweight presentation model can be linked to other models which then give formal meaning and semantics to these simple tuples. Of most interest here is the relationship to user actions to complete tasks and the availability of the widgets and their behaviours in different modes. This is described in a presentation interaction model (PIM) which is a state transition diagram with each state representing the presentation model of a mode and transitions showing the behaviours which enable a user to switch between these modes.

From these formal models of the device we can derive interaction sequences. An interaction sequence is the set of actions and interactions that a user performs with a given system to complete a task (Turner, Bowen, & Reeves, 2017). As such, they describe actions a user undertakes for a given task from a given device state. For example, if the device is in the “SetVolume” mode with a current volume value of “0”, then setting the volume to be infused to 10ml might be represented as
“PressUpButton[10]” which is shorthand for “PressUpButton” listed ten times (Bowen & Reeves, 2008). Interaction sequences can be generated automatically from the device models and can be used to represent optimal paths of actions (as in the set volume example here) or may include additional, arbitrary or erroneous actions. Such interaction sequences are, therefore, a type of task model (describing the steps to complete a task) but they are sequential rather than hierarchical and at the lowest level of abstraction.

These types of descriptions are model-specific in that they are explicitly tied to the detail of the formal model. Task models, however, can also be device-specific, where they describe the task as it pertains to a specific device (which we discuss next) or they may be independent of both of these (i.e., generic) and pertain solely to the goals of the user in terms of what they want to achieve without the low-level details of how.

Generic task models can be used to inform device design because their inclusion in a typical user-centred design (UCD) approach means that the formal models derived from UCD artefacts have this information embodied within them. That is, a prototype or storyboard created to examine initial design ideas is partly based upon the user tasks (as well as requirements, guidelines, safety regulations, etc.).

Model-specific task models, such as interaction sequences, can be used iteratively to help refine design artefacts. For example, our interaction sequence step “PressUpButton[10]” (Bowen & Reeves, 2008) described above may lead to a design evolution where a long press on the ‘Up’ button increases a value in increments of 10. This leads to shorter interaction sequences for some tasks, which might be seen as better for user experience and usability.

We have introduced a number of different models and types of models here. In Figure 1 we described how different categories of models could be related to each other and derived from each other, in Table 1 we give a summary of the introduced models and show how they fit into this structure.

**Table 1: Models used in different parts of the processes**

<table>
<thead>
<tr>
<th>Formal Models of Medical Devices</th>
<th>Technical Competence Models for Numeracy</th>
<th>Task Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentation models of device interfaces</td>
<td>MDC-PS (from Fig. 2)</td>
<td>Based on user tasks with devices</td>
</tr>
<tr>
<td>Interaction sequences of device use</td>
<td>Based on user actions and knowledge</td>
<td>Numeracy tasks</td>
</tr>
</tbody>
</table>

**Task models of medical devices**

The syringe driver in Figure 3 is a CME Niki T34 (henceforward referred to as the “T34 syringe driver”). It is used to deliver a pre-determined amount of medication from a syringe to a patient over a pre-defined period of time. The device has eight buttons that the user can interact with, as well as a small screen which provides information and feedback. In order to set up medication delivery the user needs to undertake several different tasks, some using the device (inserting the syringe, setting up the
dosage rate, etc.) and some independently from the device (calculating correct volume and time according to the prescribed dose, getting the medication, etc.).

Each of the tasks can be combined into a single action, “Set up infusion”, which can then be broken down into hierarchical steps in a typical task analysis fashion. So, we can either create a model for the generic task that a user would perform with this type of device (device-independent) or specialise it for this actual device (device-dependent). If we again consider the task of setting up an infusion, this can be decomposed into the following 5 sub-tasks:

- turn on syringe driver
- insert syringe into driver
- set volume to be infused
- set time for infusion
- start infusion

There are a number of assumptions made before this task can be carried out which are reliant on previous tasks (such as selecting the appropriate syringe type and size and drawing up the medication) having been successfully completed. A full task model would include all of these, but for now we focus on just the task actions relating to the medical device, which is typical when using such models as part of interactive system development and reasoning.

All of the steps can be broken down further and Steps 3-4 can include both iteration and repetition between the steps. There are also temporal relations between steps (some steps must be completed before others, it is possible to go back to a previous step, etc.). Here we use the CTT notation (Paternò et al., 1997) to describe some of these properties; this is done for convenience as CTT includes the relevant operators to describe these properties as well as an editing tool to create the hierarchical models (Mori et al., 2002). However, there are several other notations that could be used and we do not propose that one particular hierarchical notation is necessarily more suitable than any other.

![Figure 4: Task model for set up and start infusion](image)

Figure 4 shows the top level view of these tasks. The >> operator signifies a sequence such that the first action must be completed before the next can begin. The |={| signifies that it does not matter in which order these actions are completed, and the * represents iteration. The tasks at the second level of the tree can be further decomposed, for example, in Figure 5 we see the detail of the sub-task required to set the time of the infusion.
The task models shown here focus on user actions only. That is, they do not depend on the particular device or implementation. If we were to specialise the models for the T34 syringe driver, shown in Figure 3, for example, the “Set up infusion” and “Set time” tasks would be represented by the task models shown in Figures 6 and 7.

We can see that the “Set up start infusion” model of Figure 6 differs from the generic task model of Figure 4 in two ways. Firstly there is now a defined ordering between the “Set volume” and “Set time” tasks, as the T34 syringe driver mandates this order. Secondly there is an additional subtask, “Confirm rate”, which again is an action that must be performed when using this particular device but which may...
not be expected by a user. This mismatch between the model-dependent task model and the device-dependent task model indicates that further work is required.

The formal models describe an intended device based on user tasks and subsequent design decisions. If the implemented device differs (as is made apparent by the different task models) then we should identify why/how the changes occurred (were they intended or based on additional requirements not included in the model, for example?). They also suggest a mismatch between user expectations (as embodied in the UI design models) and the actual implementation. Now that it has been identified we might use it to inform user training or to warn of potential user error.

Such comparisons can also be useful when considering multiple devices with similar functionality (or the same devices with different firmware). If defined orderings of actions differ between different instances of devices, this is an area which may also lead to confusion or user error and so enables us to flag a potential problem. A device-specific task model can also be decomposed down to the level of the interaction sequence, so the lowest nodes on the tree represent interactions with actual widgets, such as “press upButton”. This allows us to start combining the interaction sequences of device models with model-specific task models to ensure that they are consistent.

In this work we also wish to consider a comparison of task models for devices with models of user intentions based on numeracy skills and technical competence. We discuss this next.

**Task models and numeracy/mathematics education**

There is an extensive research literature on mathematical task design (Watson & Ohtani, 2015), likewise on mathematical modelling (e.g., Galbraith, Henn, & Niss, 2007), but the literature on ‘task models’ per se relates to human-computer interaction in engineering and computer science (e.g., Paternò, 2001) rather than mathematics or numeracy education. We suggest that bringing insights from these fields together could benefit both. In particular, we suggest that such an integrated approach could improve our understanding of the numeracy demands of various user interface scenarios and the best way of ensuring that users can meet these demands, including through both education and training of users and improved interface design of devices.

Our focus is on user interfaces - and users interfacing - with safety-critical medical equipment involving digital inputs and/or reading, recording and interpreting of outputs by users, on the education and training required for this to be done competently, i.e., efficiently, effectively, and, above all, safely, and on the implications of this for the design of device interfaces.

In so doing we are bringing together research and ways of thinking developed in different academic disciplines and professional domains, with very different relationships to - and conceptions of - the notion of task modelling. In fact, task modelling in the sense outlined above has not hitherto been used in mathematics education research as far as we are aware. The nearest cognate concepts used in the mathematics education field are mathematical modelling and task design.

Mathematical modelling is the process of applying mathematics to a real-world problem with a view to understanding it (Niss, Blum, & Galbraith, 2007) while task design refers to the design of mathematical tasks for educational purposes. As such, as the editors of a recent book detailing an International Commission on Mathematical Instruction (ICMI) study on task design state, “Task design is at the heart of effective mathematics teaching and learning” (Watson & Ohtani, 2015, p. vi). However, “Despite the recent growth spurt of design studies within mathematics education, the specificity of the principles that inform task design in a precise way remains both underdeveloped and, even when somewhat developed, under-reported.” (Kieran, Doorman, & Ohtani, 2015, p. 74). Meanwhile, a recent review of the state of the art in mathematical modelling notes its “local” focus, eschewing input from other disciplines:
In research into the teaching and learning of mathematical modelling there is a strong emphasis on developing “home grown theories” where the focus is on “particular local theories” such as the modelling cycle and modelling competencies rather than general theories from outside the field (Geiger & Frejd, 2015).

(Stillman, 2019, p. 6)

We hope that tool models may provide such a theoretical (and practical) input from outside the field of mathematics education.

We suggest that meaningful mathematical tasks model activity in the real world in an authentic way (Palm, 2009; Weeks et al., 2013). In this case that means taking into account the various ways in which nurses engage with the mathematical demands of the digital interfaces they encounter in their professional practice. This variability has implications for the specification of the relevant task models. The competence model presented in Figure 2 exemplifies this approach with regard to MDC-PS.

As well as being authentic, we also suggest that such tasks should be mathematically-rich and pitched at an appropriate level of challenge for the learner. The principles of MDC-PS described above have been embodied in an e-learning environment called safeMedicate® which has been developed by Authentic World Ltd. This contains authentic clinical dosage calculation problems and supports the development and assessment of competence in dosage calculation problem solving within five skills-based modules. We present examples from these problems to demonstrate the use of task models as a mechanism for structuring the steps and activities users are required to undertake to complete the problems, which therefore represents the steps required to perform the task in a real clinical setting.

Figure 8 shows a screenshot of one of the example problems from the ‘Advanced Injectable Medicines Therapy, Continuous Infusion’ module.

![Figure 8: Screenshot of example problem from safeMedicate®, ©Authentic World Ltd.](https://www.safemedicate.com/)

---

3 https://www.safemedicate.com/
The problem describes the task of setting up a syringe driver to deliver the described medication. However, before the user can actually perform the set-up of the syringe driver there are a number of steps they must complete first, such as checking all of the details to ensure that the patient, prescription, medication, etc., are correct and match with each other. Figure 9 shows a partial task model for these initial steps (note that some of these can be decomposed further if required).

![Figure 9: Task model for initial steps to set up infusion](image)

Figure 9: Task model for initial steps to set up infusion

Figure 10 shows the task decomposition for the ‘Calculate dosage’ task. Again, this is based on the problem example from safeMedicate® and Figure 11 shows the model answer for completing this correctly, which forms the basis for the task model.

![Figure 10: Task model for calculating dosage](image)
Figure 11: Model answer screenshot from safeMedicate®, ©Authentic World Ltd.

Figure 12: Set up equation task model for specified task

Figure 13: Task model for calculating dosage for given example
The activities described in both the task models based on the numeracy example (Figures 9 and 10) and those from the device models in the previous section (Figures 4 and 5) link directly to the numbered steps within the competence model section as follows:

- (1) is required to check the information and links to the steps under the “Check Details” subtask
- (1) is required to set up the equation with the correct values and calculation steps
- (2) is required to evaluate the equation
- (3) is required to set up and start infusion

We can, therefore, identify a relationship between the two domains, where fulfilling tasks described in device models depends on competencies outlined in the MDC-PS. The actual relationship depends on the level of analysis (how far we decompose the sub-tasks into smaller steps) but we can see how it starts to become possible to identify errors or mismatches in beliefs, behaviours and requirements as we start to bring the two domains together via the models. We discuss this further next.

**Combining and comparing task models**

Now that we have described how to generate task models from both interactive system models and numeracy education tasks, we can consider how these might be used together. In Figure 1 in our introduction, we state that we want to use the comparisons of these models to: inform device design; inform user education; identify potential errors. In addition, our longer term goals are to consider how mappings between models, as well as comparisons, can be informative. Here we begin this process by discussing how we might compare and combine the two sets of models in a useful manner, i.e., one that is productive of safe, efficient and effective clinical practice.

The task models of Figures 9 and 10 represent the task described in the safeMedicate® example of Figure 8. We can, of course, decompose these further into specific (or specialised) steps for the actual example, in much the same way as we considered Figure 4 as a generic model that could be specialised for a specific medical device (the T34) of Figure 6. If we expand the “Set up Equation” subtask with these specific details we may generate a task model such as that of Figure 12.

We can do the same for the “Evaluate equation” model by adding the relevant detail from the example task, although we omit the actual model here for brevity. The higher level model of Figure 10 is now specialised, as shown in Figure 13.
In order to build a complete task model to include the setting up of the infusion device (i.e., completing a specialised version of Figure 9 we can do one of three things:

- Build the sub-task model from the problem example by inferring the steps required (using the sample device picture included in Figure 8).
- Use the generic “Set up” model of Figure 4 and add this as a sub-task to “Set up Continuous Infusion” which is dependent on the completion of the “Check Details” and “Calculate Dosage” tasks.
- Most usefully, we can use the specialised “Set up” model for whichever actual device is being used to deliver the medication (which we assume here for convenience is the T34 syringe driver) as the sub-task. If we do so and create the model shown in part in Figure 14 (where the ... replace omitted sub-tasks) we are immediately able to identify a mismatch which could lead to potential user error.

The “Calculate dosage task” is defined in terms of millilitres per hour (ml/hr), but the “Set time” task is defined in terms of minutes. If the user is unaware of this difference they have the potential to enter the calculated value without first converting to the units of the medical device. With this potential problem identified we could use the information to ensure that such a possibility is highlighted within the numeracy education and device training, i.e., the knowledge identifies a potential error and we use it to inform user education.

Conversely, we could use this information to inform device design. If we were using the combined task models during the development process we could identify the mismatch that occurs and design the device such that it enables the user to enter the values as calculated but also define the units (ml/hr) of their values. The device could then perform the conversion to whichever values it requires for its own delivery calculation. Even better, if we are using the models in initial design and prototyping processes it might suggest that the device itself could be used to evaluate the equation with the user entering all of the values and units from the equation set up directly into the device. This has the added advantage of removing any additional calculation device (such as a pocket calculator or phone calculator) which might be used to evaluate the equation and which has the potential to introduce a whole new set of errors (see Thimbleby, 2000; Thimbleby, 2015 for a full discussion of this problem).

Discussion

In the introduction we introduced our wider research goals with the following questions:

- How can we use formal models of medical devices in conjunction with task models to improve the design and development process?
- How can we use task models to identify mismatches in user knowledge and education with device usage?
- How can we use task models of medication calculations and technical competence to inform user education and the design of medical devices?
- How can we use comparisons of task models of devices and medication delivery to identify potential use errors?

We now discuss how these have been addressed in this paper.

We began by describing how generic task models can be used as inputs into formal models of interactive systems through their incorporation into informal design artefacts. While the use of task models in UCD is typical, we gain additional benefits from the use of formal models which describe the informal design artefacts as this ensures the informal inputs are captured within such models. By then deriving interaction sequences from the formal models we have shown how such model-specific task models can be used to improve designs and prototypes. This satisfies Step 1 in our overview diagram of Figure 1 and answers the first question above.
Task models of specific devices, such as the Niki T34 syringe driver described above, can be compared to the more generic models which enables the identification of potential mismatches in user expectations and actual behaviour of the device. Medical devices can have a variety of different number entry types (from simple up/down arrows to full numeric keypads). They also exhibit a variety of different behaviours regarding minimum and maximum value input roll-overs (what happens at ‘0’ if user tries to decrease the value) and positional displays which change according to whether or not decimal points exist in displayed values. These are often the cause of number entry errors, and so being able to identify the potential for some of these to occur in the manner suggested enables us to both improve design as well as provide better education by identifying to users that such differences exist. This partially answers our second research question.

We show how task models can be generated from numeracy education examples which represent authentic tasks and then describe how we can combine and compare these with generic, model-dependent and device-dependent models (Steps 2 and 3 in the overview diagram). This enables us to identify potential user errors which can be used to inform both device design and user education and partially answers our third and fourth research questions. We also show how the MDC-PS competence descriptions are related to the generation and use of the task models, which strengthens the relationship between our two domains.

Bringing together the two domains in the manner described has the potential, therefore, to provide benefits to both. The formal models and device models can suggest areas that should be included in the technical competence and numeracy education. Likewise, the task models of the authentic numeracy tasks can indicate improvements or enhancements of the medical devices. Although we have introduced the idea of using such models to support users switching between different types of device (which may have subtle differences in how they are used), we have not elaborated on this here. Similarly we have not discussed how the combined models may be used to consider the use of multiple devices for a single patient. We leave these matters for future work, however, from these initial examples we believe we have demonstrated the applicability of our methods in this area.

Conclusions and future work

In this paper we have laid the groundwork for a closer integration between software and device models used to improve design and use of safety critical medical systems with numeracy education in and for the clinical context.

We have shown how task models of user goals (which we call generic task models) can be used as inputs to formal models of interactive systems, such as medical devices. We have also shown how task models can be derived from such formal models (model-dependent task models) and also specific devices (device-dependent task models). Task models used in this way can be used to improve design and identify and mitigate the effects of potential user errors.

We have also shown how task models can be created from numeracy education examples which describe knowledge-based tasks for medication delivery to ensure nursing staff have the necessary skills to perform medication dosage calculations and administer medication. The task models can then be used in conjunction with model-dependent and device-dependent models to inform user education and identify potential user errors.

Our main contributions here are demonstrating that by expressing properties of two different domains (interactive system modelling and numeracy education) in a common language - task models - we are able to compare and integrate models. This allows us to partially answer all of the research questions posed in our introduction. It also provides the basis for our further research which will enable us to fully answer all of these questions.
In order to extend this work further we can now begin to consider deriving algorithms for traversing the task models in order to automate the process of combining and comparing the models. This may also require some ontological mapping to resolve naming differences and automate the understanding needed to identify types - such as units of time and measurement, etc.. We also wish to investigate further how different types of number entry (five key interfaces vs. numeric keypads, for example) may lend themselves to calculation competence better than others and whether this can be identified from the task model comparisons.

We believe task models may provide the solution to the problems outlined above since they can provide a link between the interactive system design and the numeracy demands, practices and associated education and training. Tasks are fundamental to both in terms of ensuring devices are designed to correctly support tasks in a usable fashion and ensuring healthcare personnel can complete specific tasks relating to medication calculations. In particular: formal models and device models may suggest areas that should be included in numeracy education, especially for the development of technical competence; task models of authentic numeracy tasks may suggest improvements or enhancements of medical devices.

References


Gottlieb, L. N. (2012). Nurses: The first and last line of defence for not only patients but physicians as well. Canadian Journal of Nursing Research (CJNR), 44(2), 3-6.


Welcoming *Articles about Practice* to ALM-IJ

Lynda Ginsburg
Rutgers University, USA
< ginsburg@scarletmail.rutgers.edu >

**Context**

As ALM conferences have been held in various countries, local practitioners have attended, with many making presentations on various themes such as: their instructional practices, activities they have created or adopted from scholarly reading, and implications of national policies for practice in adult education. At this juncture, ALM would encourage practitioners to share their experience through a published article.

Since 2005, the Adults Learning Mathematics – An International Journal (ALM-IJ) has been publishing research articles about adults learning mathematics at all levels. The articles represent a variety of theoretical frameworks, mathematical content topics, data sources, learning environments, learners’ profiles, and related discussions of policy or practice issues.

The Editorial Team of the Journal, with the agreement of the Trustees of the Journal’s parent organization, Adults Learning Mathematics, has decided to provide further opportunities to include Articles about Practice in the Journal. This move is in tune with the history and philosophy of our organization – and we hope it will benefit the adult numeracy field as a whole.

From the formation of ALM as a research forum in 1993, practitioners have been attending and participating in the annual conferences, as can be evidenced in the Conference Proceedings available on the website https://alm-online.net/alm-conference-proceedings/. In fact, one of the three or four days is typically focused on practitioners’ contributions. Practitioners work in a variety of environments, including school and community-based educational programs, workplaces, prisons, family literacy programs, family numeracy activities and civic settings. Even though their travel budgets are often limited and rarely subsidized by their institutions, practitioners have made presentations that are significant contributions to the field.

The mutual respect between ALM researchers and practitioners

Such an inclusive environment is quite unusual for many research conferences. There may be a few reasons why ALM has historically been so welcoming to practitioners at conferences, as Trustees, and even as ALM office holders, when other research associations appear to be less so.

First, many ALM researchers have themselves taught adults in non-tertiary mathematics classes, or in catch-up support courses in tertiary institutions. Sometimes, the ALM researchers came to their graduate work following their own adult education teaching, others teach adults as part of their research mission, while yet others begin their adult education teaching following research careers. In all these cases, the researchers have come to know and respect practitioner colleagues as skilled, knowledgeable and possessing expertise.

Second, the learners of interest in adult mathematics education are often of similar ages to their teachers and the researchers. And, as adults, the learners bring knowledge from their own lives, work, family and civic experience that is valued and explored in classrooms and in research studies. This represents a different dynamic between researchers, teachers and learners than the situation between...
researchers and educators who are the adults and those they study or teach who are younger – the elementary, secondary, and even typical university students.

The relationships between researchers and practitioners are further enriched as they work closely together, collaborating in research studies. Practitioners invite researchers into their mathematics education classes in the variety of environments in which they work.

For all these reasons, those with an interest in how, when, and where adults learn mathematics will benefit from opportunities to hear directly from practitioners through ALM-IJ articles.

What counts as an Article about Practice?

Practitioner articles are meant to share the perspectives and provide accounts of the experiences of adult mathematics educators with the larger Adults Learning Mathematics community. Such articles are grounded in the experience and day-to-day realities of practice and would be of interest to other practitioners, researchers and professional developers. Particularly important is to include a description of the context, that will be comprehensible to the Journal’s international audience.

Practice articles could be based on descriptions of and reflections on classroom experiences; particular mathematical content, instructional strategies and/or materials; a professional development experience and its impact on classroom instruction, or adult learner assessment issues.

Four descriptions of appropriate journal Articles about Practice are described here, but these are meant to inform, not limit, ideas for articles.

1. An Article about Practice might describe a practitioner’s own experiment or action research concerning a change in practice, with the rationale for making the change, and then the practitioner’s observations of results from the change, supplemented where possible by evidence of changes in response, or changes in engagement, from learners.

2. An educator might be asked to develop a curriculum or resources for a particular group of adult learners, perhaps in a workplace setting. An article might describe the challenge, the development of the materials, the implementation process with the adult learners, and the assessment of the resulting learning.

3. A practitioner might pose a problem of practice (e.g., why do my learners struggle with ...?) with a description and evaluation of attempted strategies to ameliorate the problem.

4. A practitioner might comment on how a particular ALMIJ research article or other resource had an impact on the thinking and practice of the practitioner, perhaps supplemented by a description of a subsequent in-class “experiment” that confirms or challenges the research findings.

What is the format of an Article about Practice?

Research journal articles typically have a structured format that includes a statement of the problem, research questions, a review of relevant research, a methodology section, findings, discussion and implications.

Articles about Practice would not likely follow this traditional research format, but would likely include the following components as appropriate:

- Rationale for why the particular topic, method or strategy is important and/or relevant to adult mathematics/numeracy educators and researchers
- Description of the adult learners and the teaching and institutional context
- Description of the implementation
What is the submission process for Articles about Practice?

Practitioners’ articles should be submitted electronically to the ALM-IJ Editorial Team by emailing editor-ij@alm-online.net with the article as a Word document attached. A second version of the article with the author(s) name and any other identifying information removed should also be attached. This anonymized version will be used to send to reviewers to uphold the anonymity of the author(s) during the review process. If there are pieces of scanned student work or other figures, they, too should be attached if they cannot be easily embedded within the documents.

The goal of publishing Articles about Practice is to share interesting, well-contexted and high quality information to improve teaching and student learning in adult mathematics. Thus, each submitted Article about Practice will undergo a double-blind peer review process. The review process and resulting feedback to the author(s) aims to provide valuable comments and critique that should be used to improve the quality and impact of the article. This is a similar process to that applied to each research article submitted to, and ultimately accepted by, the ALM-IJ.