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Objectives

Adults Learning Mathematics (ALM) – An International Research Forum has been established since 1994 (see www.alm-online.net), with an annual conference and newsletters for members. ALM is an international research forum that brings together researchers and practitioners in adult mathematics/ numeracy teaching and learning in order to promote the learning of mathematics by adults. Since 2000, ALM has been a Company Limited by Guarantee (No.3901346) and a National and Overseas Worldwide Charity under English and Welsh Law (No.1079462). Through the annual ALM conference proceedings and the work of individual members, an enormous contribution has been made to making available research and theories in a field which remains under-researched and under-theorized. In 2005, ALM launched an international journal dedicated to advancing the field of adult mathematics teaching and learning.

Adults Learning Mathematics – An International Journal is an international refereed journal that aims to provide a forum for the online publication of high quality research on the teaching and learning, knowledge and uses of numeracy/mathematics to adults at all levels in a variety of educational sectors. Submitted papers should normally be of interest to an international readership. Contributions focus on issues in the following areas:

- Research and theoretical perspectives in the area of adults learning mathematics/numeracy
- Debate on special issues in the area of adults learning mathematics/numeracy
- Practice: critical analysis of course materials and tasks, policy developments in curriculum and assessment, or data from large-scale tests, nationally and internationally.

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Adults Learning Mathematics – An International Journal

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Editorial

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In this special issue, we aimed to collect research and practices that focused on adult learning in mathematics in online spaces. We wanted to see successful practices of adults engaging with formal and informal mathematics in several different contexts. The main objective of the special issue was to offer a platform for the teachers around the world to share their experiences to improve adult numeracy and pedagogies. Submissions were received from the UK, Brazil, Australia, and Canada. Unfortunately, one of the papers was recently withdrawn. A systematic literature review provided an overview of the research in the last five years; a second identified possible adaptations of teaching mathematics online from a family learning context; and the third used a theory for teaching *building thinking classrooms* to create guideposts to teach adults in a fully online setting.

Linda Galligan and Taryn Axelson's literature review identified three streams: adults learning mathematics online (including attitudes to technology and online learning; and the impact of online learning on student success and engagement), teaching mathematics online, and the impact of Covid-19 on online mathematics education (on student learning, educators, and pedagogical innovation). They noted the dearth of research in online learning in the context of adult learning mathematics and recommended future research focus on tailored approaches to the learning needs of adults; approaches that ensure no disadvantage to adults or to learning outcomes; and ways to effectively bridge educational disadvantage including the use of open access resources.

Beth Kelly, Mary Devlin, Tessa Giffin, and Linda Smith investigated the effectiveness of an intervention on family learning online during lockdown in South London. They used an action research approach and found opportunities and challenges for both the 20 mainly black and minority ethnic parents (all but one were women) with their 48 children, and the three family learning tutors dealing with new and often unfamiliar technologies. The research was situated within a Family Learning program where a variety of courses were offered for the first time online. Their findings were grouped into three themes: a focus on mathematics (including parents' attitude to mathematics and reflection on their own knowledge); the families' experiences (mainly positive and enjoyable, but they identified challenges); and the tutors' reflections (positive but with social, technical, and emotional challenges). They highlighted the need for professional development for tutors and suggestions for further research into the digital ecology of family learning

In the final article, Judy Larsen and Peter Liljedahl conducted a thematic analysis of the Building Thinking Classrooms (BTC) practices to identify, through a theory *for* teaching, the core guiding principles to highlight aspects that were important to teaching online, to assist in the design of effective and intentional learning settings in the adult mathematics context. The thematic analysis they conducted was on a data set from thousands of previously observed micro-experiments in the K-12 setting. The results produced six principles of practice: *decentralize control*, give them something to think about, do first, *diversity is a strength, mobilise knowledge*, and assessment as communication. These principles of practice were then tailored to the online context for pre- and in-service teachers and three principles were further explored in this article (in italics). The principles developed can assist educators in an adult learning mathematics setting to determine how best to facilitate learning.

We hope that you will enjoy the readings, and see you in the next volume.

Galligan, L. & Axelsen, M. (2022). Online learning in adults learning mathematics: literature review. Adults Learning Mathematics: An International Journal *16(1), pp.6-19*.

Online learning in adults learning mathematics: Literature review

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Abstract

This systematic literature review provides an overview of research and practices that have occurred over the last five years in the realm of adults learning mathematics through online learning. It includes research in the tertiary space, much of which has focused on university education. Systematic literature reviews are a method of making sense of large bodies of information and are a method for 'taking stock' and understanding of the evolution of a field of inquiry. The review found that there have been three main streams of research in the literature on adults learning mathematics online. The research in these three streams is summarised and discussed. From the review it is apparent that in the context of adults learning mathematics, there is a dearth of research on online learning in general; while there are studies that have examined the use of certain technologies, there is less of a focus on the online learning context for adults learning mathematics. Recommendations for areas of research are provided.

Key words: online learning; online technologies; adult learning, mathematics

Introduction

E-learning or online learning first emerged in the late 1980s and early 1990s when computers were increasingly being used in the home (Li, 2018). Among the first tertiary institutions to begin online learning during this time were the Open University in Britain, the University of Phoenix in the USA (Kentor, 2015) and the University of Southern Queensland in Australia as a mixed mode university (Postle et al., 2003). Since its emergence, online learning has been made increasingly possible and accessible not only by the advent of the Internet and World Wide Web but also by the high-speed internet connections that have typified internet usage in the last decade.

Since the emergence of online learning, its adoption has been rapid and transformational, leading to a paradigm shift in learning and teaching practice in higher education and vocational training institutions. For many educators, online learning is seen as a way to enhance and improve the student learning outcomes while battling shortages in resources, facilities and equipment (Castro & Tumibay, 2021). Resources that are freely available and accessed online, called open access resources or open educational resources, have been found to be useful in providing content, practice problems, grading, and feedback to students. Khan Academy, for example, has been a leading educational resources repository, with mathematics its most popular subject. Other open access resources in mathematics include *Matific* and *Geogebra*, both of which use interactive technologies and resources that support the learning of mathematics.

From a student perspective, online learning has led to a democratizing of educational delivery. At the same time, however, it has also led to a digital divide. On one hand, due to its potential for providing more flexible access to content and instruction at any time and from any place, online learning has become synonymous with accessibility, affordability, flexibility, and life-long learning. For many learners, online learning makes the ability to engage in education possible as it enables them to integrate education into their lives rather than interrupting their lives for an immersive experience. One group of learners for which this is particularly the case is adult learners.: For this group of learners, who are more likely to have work and family obligations to balance alongside attaining a degree, online learning provides the flexibility they require to engage in more advanced levels of study (Francis, Wormington, & Hulleman, 2019; Meisami, 2020). However, research has also recognised that online learning may be more beneficial for some types of students compared to others (Francis et al., 2019). This means that as well as democratizing of educational delivery, online learning has also led to a digital divide. Inequalities in the ways different students access and engage with online learning occurs due to such factors as network coverage, device type, time of day, socio-economic status, age, educational background, and digital competence and experience (Azıonya & Nhedzı, 2021; Dhawan, 2020; Eynon, 2009; Hillier, 2018). Thus, as recognised by Dhawan (2020), while online learning can help in providing inclusive education, online learning systems, pedagogies and practices need to be developed in such a way that ensures no student is disadvantaged by the use of online learning as a result of, for example, their location, social class, and ethnicity. The need to continually consider this digital divide also means there remains the need to be highly cognizant of the impacts on students of the technologies, systems, pedagogies, and practices used on online learning and teaching.

Although online learning was already experiencing a rapid uptake by educators and students before the Covid-19 pandemic, for many universities and vocational education providers, the pandemic accelerated the move to online learning. Large numbers of lectures were quickly moved online as online learning became a panacea in the time of crisis. This quick transition meant that educators who may have had little or no experience of teaching online were suddenly tasked with adapting their classroom-based teaching approaches and materials to the online space, and they had to quickly learn how to teach and engage students in an online environment (Albano, Antonini, Coppola, Dello Iacono, & Pierri, 2021; Livy, Muir, Murphy, & Trimble, 2021).

For many educators, this rapid move online proved challenging, and while in some cases the impact was ‘bad online teaching’, it also challenged educators and researchers to engage with learners using Information and Communications Technology (ICT) tools and methodologies and it stimulated new ideas and new ways of approaching teaching and learning. It also prompted educators to re-examine their teaching (e.g. Albano et al., 2021; Brunetto, Bernardi, Andrà, & Liljedahl, 2021; Maciejewski, 2021) and has opened up a whole new vista of education possibilities (Trenholm & Peschke, 2020). Furthermore, while concerns for student equity was one of the reasons some educators resisted the move to fully online learning formats prior to the Covid-19 pandemic, during the emergency transition to online learning, strategies related to an ethos of care were adopted by educators to mitigate and address equity issues, including those of flexibility, reducing coursework to essential content, and personalization (Goin Kono & Taylor, 2021).

As the adoption of online learning has been rapid and transformational, and is constantly leading to evolutions and innovations in learning and teaching practice, it is useful to regularly stop to ‘take stock’ of the innovation and research that has occurred to identify trends and innovation in practice, impacts on and changes in student experiences, and potential areas where learning and teaching practice needs be improved. It is also cognizant to pause to reflect regularly. First, the sector needs to ensure learning and teaching practices address contemporary students’ learning needs and lead to deep learning rather than an ‘illusion of understanding’ (Schwartz, 2013, p. 1). Second, the sector should explore and articulate how learning methods in a discipline have evolved with changes and developments in online learning technologies and online learning trends. To contribute to our understanding of how online learning has impacted on how adults learn mathematics, this paper provides a systematic literature review of the research from the last five years on online learning in mathematics, with a focus on adult learners and adult learning contexts.

Methodology

Systematic literature reviews are a method of making sense of large bodies of information and involve an interpretation of a selection of documents on a specific topic that optimally involves summarization, analysis, evaluation, and synthesis of the documents (Petticrew & Roberts, 2016). Singh and Thurman (2019) suggest systematic literature reviews are a useful method for understanding of the evolution of a field of inquiry, while Tondeur, van Braak, Ertmer, and Ottenbreit-Leftwich (2017) propose the advantage of such a review is that it can produce a map of the 'bigger picture' of a specific topic or area of research. The purpose of this systematic literature review is to explore and highlight the innovations that have occurred in online mathematics education, as well as changes in both practice and the contemporary learning needs of adult learners.

The researchers collected papers relating to adults learning mathematics using online learning or online learning technologies. Journals that focus on mathematics learning (in higher education) and adults learners were of particular interest, including Educational Studies in Mathematics, Mathematics Education Research Journal, International Journal of Mathematical Education in Science and Technology, and Adults Learning Mathematics; however the search terms 'adults' and 'online learning'; and 'mathematics' were also used to find related papers published in other journals, such as those in with a focus on teaching and learning or technology use in higher education.

In determining what constituted 'online learning', it was important to define our understanding of online learning. This is because, despite the central role online learning now plays in the higher and vocational education space, authors and scholars often use the term 'online learning' to mean very distinct, and sometimes contradictory concepts, and the term has a range of meanings attached to it (Singh & Thurman, 2019). For example, it may be defined as learning taught live via technology, or hybrid learning with online content and practice problems, or fully online [perhaps even self-paced] with teachers available to help, or asynchronous learning with recorded teaching lectures.

In this paper we understand online in terms of a definition proposed by Singh and Thurman (2019). From their systematic literature review of definitions of online learning (1988-2018), Singh and Thurman (2019) suggested that online education may be defined as "education being delivered in an online environment through the use of the internet for teaching and learning. This includes online learning on the part of the students that is not dependent on their physical or virtual co-location. The teaching content is delivered online, and the instructors develop teaching modules that enhance learning and interactivity in the synchronous or asynchronous environment" (p. 306). We also acknowledge there may be two streams to online learning: one in which learning is undertaken fully online; and a second in which online learning technologies are used as a tool for learning and teaching.

Based on the results obtained from the literature search, the researchers found 31 articles that met the criteria; that is, they were about adults learning mathematics though online learning. While the literature search was limited to the last five years (since 2016), most of the articles (20) were published in the last two years (2020, 2021). Of these, half (10) were written in response to the Covid-19 and the rapid move to online education as this move led to much innovation and practice change in online mathematics instruction and learning. Thus, in undertaking a systematics review of the literature, we identified three main streams of research and the papers are reviewed within these three streams:

- Adults learning mathematics online;
- Teaching mathematics online; and
- The impact of the Covid-19 pandemic on online mathematics education

Systematic Literature Review (2016-2021) about online learning and adults learning mathematics

Adults learning mathematics online

One of the consequences of the increased accessibility of online education and the resultant democratization of higher education has been an increase in the number of adults undertaking tertiary study (Barr, 2016; Talmage, Mark, Slowey, & Knopf, 2016). As mathematics-based courses are part of the curriculum for many programs and disciplines (e.g. in nursing, paramedicine, business, psychology, engineering, computing), many adult learners undertaking tertiary study will be required to study a mathematics-based course as part of their program. The increased prevalence of online learning in mathematics education, as well as the increasing number of adults studying mathematics online has led to concerns about how the delivery of content through online methods has affected these learners' engagement and outcomes in mathematics courses and subjects.

Attitudes towards technology, online learning, and mathematics

Data sources consisted primarily of participants' work of basic functional mathematics lessons, author's notes within a research journal and conversations with refugees, support staff, and ESL teachers. This study was conducted between January 2017 and January 2018. This period was significant because of the multiple judicial challenges of presidential executive orders made by the United States executive branch. In addition, multiple agencies within a large, Midwestern urban area with extensive networks of supporting refugees during their resettlement period provided background information on lands of the refugees' origins.

Mature students are often anecdotally thought to be more anxious about technology than younger students, to the extent that they avoid using technology; however, research has shown this is not generally the case. In their research exploring age differences in use and attitudes towards online learning and the use of digital technologies, Staddon (2020) found no difference between mature and non-mature students' attitudes overall, nor for the dimensions related to confidence and utility (students in this research came from a range of disciplines, including mathematics). While this research did find that mature students used fewer technologies than younger students and used them less frequently, the mature students had used those technologies for a longer period over their lives.

Research by Heretick and Tanguma (2021) into attitudes towards statistics among adult learners discovered older adult students found their online learning experiences in statistics to be more rewarding than younger adult learners. In addition, when their possible deficits in online technology were addressed, these older students performed as well, or better than, their younger colleagues.

For adults learning mathematics, including in the online learning mode, research has shown that challenges are often more related to the mature-aged status of the learners rather than the mode of study, especially in relation to mathematics competence and attitudes. For many mature-aged students, there can be a considerable gap in time between finishing compulsory education and beginning more advanced study, such as at university (Galligan, Frederiks, et al., 2017; Robinson et al., 2019). Engagement with mathematics may also be accompanied by apprehension and fear, particularly if previous experiences of mathematics have been negative, or if learners lack mathematics experience or are underprepared, lack confidence, feel anxious about mathematics, or have protected themselves by avoiding the subject (Cousins, Brindley, Baker, & Johnston-Wilder, 2019; Ryan & Fitzmaurice, 2017). Many mature students may also feel at a disadvantage compared with their younger counterparts who have entered higher levels of education straight from high school where they have had recent mathematics educational experience (Ryan, 2019). Some research has also found that mathematics prerequisites for entry into undergraduate programs are not enforced at many universities (Robinson et al., 2019). When students are only advised of the level of senior mathematics that is 'assumed' or given the 'recommended knowledge' for their degree, they can feel uncertain whether their current mathematical skills are enough for the tertiary study they are about to undertake.

Heretick and Tanguma (2021) suggested that for adults learning mathematics, online learning provides opportunities for the application of heutagogical principles, also known as self-directed learning. Methods based on heutagogy expand learners' control and choices over their own learning experiences. Such methods are particularly relevant in online education as it is a learning context that often requires students to guide their own learning and be self-motivated in that learning (Yen & Liu, 2009). When applying heutagogy to the online education of adults learning quantitative-based content (such as in statistics and mathematics), Heretick and Tanguma (2021) argued that it is a method that is most appropriate for adult learners who come to online study with competence but also with positive attitudes, expectations, and adaptability.

Impact of online learning on student success and engagement

Researchers have been interested in comparing the learning outcomes for students undertaking online learning with those students undertaking traditional face-to-face learning. Findings from these comparisons have delivered mixed results. For example, in a study of 1,411 learners (of various ages) undertaking mathematics courses, Francis et al. (2019) found that students enrolled in online courses received lower course grades and lower pass rates compared to students enrolled in face-to-face courses. The authors noted, however, that the students enrolled in online courses were more likely to be women, identify as an underrepresented ethnic/racial minority group, adult learners (i.e., 25 or older), and enrolled part time, all of which are considered factors that impact student success (i.e. gender, race/ethnicity, age and type of attendance; see Alyahyan & Düştegör, 2020, for a review of the literature on the factors that impact and predict student success). The authors acknowledged, however, that in interpreting their research it was important to note that the size of effects were small (Cohen's ds = 0.17–0.28).

In a study of 132 adult learners learning mathematics, Moreno-Guerrero, Aznar-Díaz, Cáceres-Reche, and Alonso-García (2020) reported finding that students studying through the e-learning method (online) were more likely to do better than their counterparts studying through the traditional expository method (direct instruction by the educator). The authors found that the use of online learning had a positive influence on the motivation, autonomy, participation, understanding of mathematical concepts, and grades of learners (compared to traditional teaching methods). The authors did note, however, that while the teaching to the e-learning cohort occurred through online methods, students had the option of attending a face-to-face teaching session if they felt they needed it; although, "hardly any [e-learning] students attended the face-to-face option" (p. 6).

Meisami (2020) examined the impact of online learning on the success of undergraduate students enrolled in a low-level mathematics course. As well as comparing differences between modes of delivery (online versus face-to-face), the author also compared whether characteristics such as age (adult students versus school leaver) had an impact on success in the course. The authors found that adult students studying in low-level mathematics courses that were fully online performed just as well as their adult learner counterparts who were studying face-to-face. They also found no correlation between age and success in the course.

In a review of the literature on fully online undergraduate mathematics instruction, Trenholm, Peschke, and Chinnappan (2019) found that while some studies reported lower student achievement in fully online mathematics courses, other studies reported finding no differences in student outcomes in fully online instruction compared to face-to-face instruction. In their review, the authors also found that students in fully online mathematics courses tended to be more dissatisfied compared to students in face-to-face instruction, and retention rates were lower. They therefore argued that compared to face-to-face instruction and fully online instruction occurring in other disciplines, the fully online modality has not worked well in tertiary mathematics instruction and suggested that more pedagogical – rather than just technological – innovations were needed.

The mixed findings emerging from research exploring how online learning has affected student engagement and outcomes (compared to face-to-face instruction) could be a result of different ways that researchers define and interpret online learning (for example, is it learning that is undertaken fully online or is it learning in which online learning technologies are used as one tool in the instruction). In addition, in many studies the definition of online learning informing the research is not well articulated, thus making comparisons across studies difficult. Research by Johnson and Mejia (2014) has also shown that success in online learning (in general, not specific to mathematics) has a relational link to time; the authors found that while online students displayed negative short-term effects for course-level performance and persistence, in the long-term there were positive long-term outcomes from ongoing engagement in online learning (including degree completion).

Teaching mathematics online

As mathematics teaching and learning has increasingly moved into the online realm, researchers have been interested in exploring how to best teach mathematics online and the impacts of this online learning on student understanding and student success in mathematics. Indeed, an ongoing issue in mathematics learning and teaching is the visual nature of mathematics, which can become problematic when instruction and learning is moved online. In a review on the use of digital technologies in tertiary mathematics, Attard, Calder, Holmes, Larkin, and Trenholm (2020) noted that concerns related to how to communicate mathematically in digital technology-enabled mediums, continue to be a recurring research theme. Such research has raised concerns about the constraints these technologies place on effective communication of mathematical language, syntax, and symbolism, critical to interactions and, ultimately, successful task completion in mathematics. The authors suggested such constraints may increase cognitive load, thus making an already challenging subject even more difficult to learn.

Since the early 1990s, when technological tools of video conferencing and audio graphics first started to emerge, they have been trialled to make mathematics ‘e-learning’ more visual (Galligan, McDonald, Hobohm, Loch, & Taylor, 2015), with varying success. Today, the use of video as an online learning tool still continues to challenge current boundaries of educational practice in mathematics education (Attard et al., 2020). Screencasts (Dunn, Loch, & Scott, 2018), recorded lecture videos (Tisdell & Loch, 2017; Trenholm, Hajek, et al., 2019), and student-created videos (Galligan, Hobohm, & Peake, 2017) are some of the ways educators deliver and discuss mathematics online.

The growth of open access instructional video repositories and their widespread use as a tool to support mathematics education has also raised a need to assess how those educational resources impact on learning. For example, although Kahn Academy has been an educational resource since 2008, and it is regularly updated and improved, studies of its impact on teaching and learning, particularly since the employment of the flipped classroom model, are still emerging. A recent investigation of the effectiveness of Kahn Academy, including for mathematics teaching and learning, has highlighted the need for more robust research into these types of resources (Yassine, Kadry, & Siciilia, 2020). While there is little research exploring adults’ use of open access resources to support their learning in mathematics, a study by Attard (2016) evaluating *Matific* as a learning resource, found that *Matific* enabled teachers to individualise learning, and that when learning is differentiated this way, students appeared to be more engaged as the content with which they were interacting contained an appropriate level of challenge. While the research examined the attitudes of primary school teachers, arguably the same principles of individualisation and differentiation could be applied in an adult learning context – this would, however, be an interesting question for researchers exploring adults learning mathematics.

While much research suggests that students are generally satisfied with such ‘online’ forms of instruction (either as a supplemental resource or a replacement for live lectures; Attard et al., 2020), Trenholm, Hajek, et al. (2019) found that a reduction in live lecture attendance coupled with a dependence on recorded video lectures has been associated with an increase in surface approaches to student learning. Surface approaches to learning are based on extrinsic motivations where the goal is to avoid failure with minimum time and effort, leading to 'surface' type learning actions such as rote learning and memorizing concepts. Alternatively, deep approaches to learning are based on intrinsic motivation, where the goal is to maximize understanding (Le, Joordens, Chrysostomou, & Grinnell, 2010). A student who takes a surface approach focuses on the concrete aspects of tasks, while a student who adopts a deep approach focuses on the meaning of the task (Trenholm, Hajek, et al., 2019). The authors concluded that regular use of recorded video lectures as the main tool for instruction may therefore reduce the quality of student learning.

The impact of the Covid-19 pandemic on online mathematics education

Since the pandemic, a volume of research has emerged examining how the pandemic and the rapid move to online education has impacted mathematics learning and teaching. This research has focused on a range of impacts and can be broadly categorised into three main streams: impacts of the pandemic and the rapid move online on student learning, impacts on educators, and pedagogical innovation in mathematics that resulted from the rapid transition online.

Research exploring the impacts on students learning in mathematics

A small number of studies have investigated the impact of the pandemic and the rapid move online on student learning. For example, Bringula, Reguyal, Tan, and Ulfa (2021) examined the challenges for 69 online college learners studying 'numerical analysis' in Manila, the Philippines during the pandemic. The authors found that the shift to online learning had a negative impact on the mathematics self-concept of learners (i.e. students confidence in their skills and abilities to accomplish certain tasks). Tyaningsih, Arjudin, Prayitno, Jatmiko, and Handayani (2021) similarly explored the impacts of the pandemic and the move to 'learning from home' on students in Indonesia. The study was undertaken with 169 higher education students studying mathematics. Using qualitative surveys to explore students experience of online learning from home, the authors reported that students experienced several constraints and difficulties that impacted on their ability to learn and participate in their learning. These included internet network constraints, costs related to increase internet use (and not having the resources for those costs), and environmental restraints (such as no room in the house to adequately be able to learn online).

Matthews, Jessup, and Sears (2021) highlighted how students of colour, who have traditionally been marginalised, continued to experience inequalities during the pandemic and this was exacerbated by the historical positioning of Black parents as obstacles to learning. The authors thus argued for the expansion of direct networks for Black parents to share, communicate, and advocate for their own needs and spaces around mathematics as a way to support the online mathematical learning of their children.

Darragh and Franke (2021) explored parent perspectives on home-learning mathematics during the lockdown in New Zealand. One hundred and sixty-four parents participated in the study, and their children (n =1 260) were from all levels of school (primary, intermediate, and secondary). The authors reported that parents were generally very engaged in the home learning of mathematics. They also found that to support their child's mathematics learning, many parents turned to online mathematics programs (open access education resources), about which they were very positive.

Research exploring the impacts on educators of mathematics

A larger number of studies focused on the impacts of the pandemic and the rapid transition to online education on educators at all levels of education (primary, secondary, tertiary). This body of research examined issues that include how educators responded to the sudden move to online learning, how they managed their teaching activities, what distance/online practices in mathematics education emerged in response to the pandemic and the push to move online and how educators experienced these practices. A common thread through many of these studies was the process of discovery – discovering new ways to teach, new ways to teach online, new ways to use technology, and importantly, new ways to teach mathematics.

For example, Albano et al. (2021) used essays composed by 44 Italian mathematics teachers from primary school to undergraduate level to explore how these teachers perceived the unexpected transition from a face-to-face setting to distance education. The authors found that the disruption of the educational setting led to teachers' discovering key aspects of the didactic system including the teacher's roles, their reflection on mathematics and its teaching, and attempts to reconstruct the didactic system in a new way.

Cassibba, Ferrarello, Musso, Pennisi, and Taranto (2021) explored how 27 Sicilian State University mathematics professors taught mathematics at distance, something many had never experienced before. The authors found that the educators started to appropriate new artifacts (pen tablets, mathematical software, e-learning platforms) to replicate their face-to-face teaching modality, with the majority (59%) finding the move online beneficial in terms of better teaching or learning.

In a study of 1719 secondary mathematics teachers, Drijvers et al. (2021) explored the distance practices that emerged during the pandemic and how teachers reacted to the experience. Results from the research showed that the use of video conferencing tools increased substantially, while the use of mathematics-specific tools that were used before the lockdown reduced. Unsurprisingly, teachers' confidence in using digital technologies was found to increase considerably during the lockdown.

Krause, Di Martino, and Moschkovich (2021) used personal narratives to show the impact of the pandemic on their goals related to teaching mathematics, as well as to examine aspects of the standards for mathematics education. Three common themes emerging from the three narratives were: developing a positive mindset toward mathematics; improving interdisciplinary connections; and considering interpersonal and collective matters beyond the individual.

Lopez, Bruun, Mader, and Reardon (2021) explored the experiences of 51 mathematics and statistics instructors before and after the rapid pivot to online teaching and found that while most educators did not change their preference for face-to-face teaching, many were amenable to using more online tools and were slowly accepting additional technological formats.

Fakhrunisa and Prabawanto (2020) explored the perceptions of 48 teachers on online mathematics learning challenges and possibilities during the pandemic. The teachers came from various educational backgrounds, including holding a bachelor's degree, master's degree, or graduating from a professional teacher education program. The authors found that educational experience was one of the factors that impacted teachers' ability to pivot to, and teach online. While the mathematics teachers generally held positive perceptions of online learning and felt it encouraged students to learn more independently, challenges they faced included their readiness and confidence in using the technologies and programs required to undertake online learning, limitations in achieving learning that demands mathematical thinking (online), and constraints in giving feedback to students.

Research exploring pedagogical innovation in mathematics

In exploring the impacts of the pandemic and the resultant rapid move online, a number of researchers recognised the opportunities the pandemic offered to usher in major pedagogical innovation in mathematics. For example, Livy et al. (2021) discussed how the Substitution, Augmentation, Modification and Redefinition (SAMR) Model can be used to adapt mathematics courses to be taught fully online. The model, developed by Puentedura (2015), was designed to classify types of tasks teachers can use to integrate technology into their classroom. In the model, Substitution refers to the use of technology to replace a task with little or no change, such as narrating a PowerPoint presentation. Augmentation occurs when the technology provides additional functions for the task, such as using an interactive quiz application. Modification occurs when significant changes are made to the learning experience via technology, such as using Google Classrooms or Zoom, and Redefinition occurs when the task is adapted and is unimaginable without the use of technology and cannot be completed without the technology.

From interviews with 120 secondary school mathematics teachers in the Saudi Arabia, Alabdulaziz (2021) explored the types of digital technology that were (and are still being) used in mathematics education during the pandemic. These included mobile technologies, touchscreens and pen tablets, digital library and designing learning objects in mathematics education, Massive Open Online Courses (MOOCs) in mathematics, and computer algebra systems (CAS) such as *Mathematica, Maple, MuPAD, MathCAD, Derive* and *Maxima*. Alabdulaziz (2021) reported a positive attitude towards the required use of such technologies, as well as a positive view towards the increased adoption of digital learning in mathematics education in the future.

Future research

As the pandemic has become recognised as a 'gateway for digital learning in mathematics' (Alabdulaziz, 2021, p. 7609), and mathematical learning and teaching will, as a consequence, be increasingly fully online, we argue that more than ever it is important that researchers and practitioners consider how to best design online learning environments to ensure optimal engagement for learning. Indeed, teaching mathematics online requires more than simply transferring learning materials designed for the traditional classroom-based learning environment to an online learning platform. It also requires careful consideration of the design and functionality of online content, including how it is sequenced, its delivery, and its ease of use – these factors have been identified as among the most influential factors determining student satisfaction, engagement, and their perceived learning in online learning (Jordan & Duckett, 2018; Kite et al., 2020; Xu & Mahenthiran, 2016).

It would be interesting to see research that explores and discusses how online learning design and content can be tailored to meet the learning needs of adults learning mathematics – and it would be prudent for such designs (and the associated research) to consider the research findings presented in this systematic literature review about adults learning mathematics online. For example, it would be interesting for research to explore and provide examples of how heutagogical principles (as suggested by Heretick & Tanguma, 2021) can be applied to online learning design in mathematics to cater to adults' learning needs, and the impact of this design on those learners.

It is also essential that future research considers how to bridge the digital divide to ensure no students are disadvantaged by online learning (particularly where it is the only study option). Such research needs to also consider the technologies, systems, pedagogies and practices used in online learning and teaching and how these may be developed to ensure equitable access for all students.

With the increasing number of adult learners entering more advanced levels of study, such as higher education and vocational education, it is important that research considers how to best meet the needs of these learners. While adult learners (or mature-aged students) no longer show any differences in attitude towards the use of educational technologies compared to their younger counterparts (Heretick & Tanguma, 2021; Staddon, 2020), challenges more related to the mature-aged status of the learners rather than the mode of study remain an issue for these learners (e.g. Cousins et al., 2019; Galligan, Frederiks, et al., 2017; Ryan & Fitzmaurice, 2017). Here, researchers and practitioners have the opportunity to explore how online learning can be used to bridge any educational gaps the learner is experiencing. Indeed, exploring how online learning and online learning design and content can be used to improve mathematics anxiety or a lack of mathematics confidence for adult learners is another relevant issue that should be explored.

Another issue of ongoing importance in research exploring adults learning mathematics online is student outcomes in online learning contexts. Findings from research comparing the learning outcomes for students undertaking online learning with students undertaking traditional face-to-face learning have been mixed. While some research has shown no difference in student learning outcomes between the modes of learning (online versus face-to-face), other research has found that students are less likely to be successful when studying online – and this is a concern. Thus, as well as continuing to evaluate student outcomes (success and engagement) when they are undertaking online learning, it is important to also explore how students best learn mathematics through the different modes (online, face-to-face) and, if there are differences in learning outcomes, how these can be addressed when that learning is moved online.

While research has considered how certain technologies may be used to enhance learning in mathematics, such as the use of screencasts and pen technologies, a wider consideration of adults learning mathematics in fully online learning contexts remains under-researched in relation to this issue and therefore a less understood area of adult learning. Thus, when exploring the use of online learning technologies, there is the opportunity to engage in research that explores how adult learners use and engage with the innovations, as well as how such technologies may be used to support the needs of adult learners.

Further understanding how adults use open access educational resources is a related stream of research. As identified by Yassine et al. (2020) from their investigation into the effectiveness of Kahn Academy, there is a need for more robust research into these types of resources. From an adult learning mathematics perspective, as adult learners often engage in help-seeking behaviours (Richardson, 2018), it would also be interesting for research to explore how adults learning mathematics use open access education resources to support their learning and how they go about seeking help from these resources in an individually motivated way. Exploring how open access resources that offer interactive learning technologies and resources can be used to individualise learning for adults is another area for research. As has been found for younger students (Attard, 2016), when learning and content is differentiated and individualised to the adult learner's required level of challenge, do adult learners become more engaged? This is indeed an important question in higher education where online engagement remains a contemporary challenge and issue.

Finally, while much innovation and change in practice emerged as a consequence of the pandemic, it is important that we do not lose this momentum in innovation and practice change. Researchers need to continue to engage with the innovation that occurred to explore long term changes in practice, ongoing impacts on adult learners, and how those innovations may be further developed and evolved.

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Family Learning Online During Lockdown in the UK

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Abstract

During the first national lockdown in the UK in March 2020 family learning sessions usually delivered in face-to-face groups were adapted to online learning encounters. This article describes research undertaken into the effectiveness of the interventions from the perspectives of parents and tutors. Using an action research approach the reflections and findings are explored to identify possible adaptions of teaching mathematics online in a family learning context.

Teaching and learning using digital platforms is a new experience for many tutors and learners who were pushed into using an online learning approach because of the strict limitations placed on physical social interactions. Not surprisingly participants and tutors experienced online learning as quite different from traditional face-to-face classrooms. The research identifies possible opportunities that can be developed through online intergenerational learning experiences. However, the findings also identify significant challenges when developing knowledge through technology, what Englebrecht, Llinares and Borba (2020) identify as knowledge developed collectively and described as 'humans-with-media'. Developing new learning experiences using digital platforms requires the ability to overcome many different technological barriers and make changes to traditional pedagogical approaches while dealing with a variety of digital and conceptual capabilities of learners. This research points to the need for professional development for tutors to foster confidence developing the new skills and the knowledge needed to sustain this different learning experience.

This research makes an original contribution to mathematics research in two ways. It discusses the intersection between the different pedagogies of family learning, online learning and mathematics learning. The research also spotlights the opinions of mainly black and minority ethnic women who are experiencing family learning online for the first time, giving voice to an often under researched group of learners.

Key words: family learning; online learning; mathematics; numeracy

Introduction

During the March 2020 national lockdown in the UK, resulting from the Covid-19 pandemic, teachers and tutors had to adapt quickly to deliver courses and workshops online. This change in teaching approach enabled family learning in a small tutors social enterprise based in London to continue with their work and to support many learners during the crisis.

These events also set up many challenges and put additional demands on both tutors and participants experiencing family learning online for the first time. The initial purpose of this research was to enable family learning tutors to continue to develop online teaching with a whole range of families, including those who might be viewed as marginalised or disadvantaged, thus helping the teaching team to better understand and respond to the participants' needs. It explores the apparently competing demands of using a family learning approach to develop skills and knowledge in mathematics through the use of technology.

This research aims to engage with adults and families to hear their views and experiences enabling the development of a more collaborative response to future online family learning programmes, including mathematics. In this article, the term parents is used to mean any adult including parents, grandparents, guardians and carers, who have direct responsibility for supporting children's welfare.

Literature Review

This short literature review focuses on the themes of family learning, learning mathematics, and online learning. We explore the characteristics of family learning and learning and teaching mathematics, describing learning that happens with and through family members. The push to online learning during the first lockdown in March 2020 was for many parents, children and teachers a novel experience, so this research explores how learning online is different to traditional face-to-face classes and how the use of digital devices affects teaching, learning and knowledge development during family learning sessions, focusing on mathematics.

Family learning

Research indicates family learning has a very broad set of characteristics encompassing learning that happens in family contexts in their broadest understanding. Family learning focuses on supporting the skills, knowledge, confidence and well-being of families by working directly with parents, carers, guardians, grandparents, siblings and the children they care for (Brassett-Grundy, 2002). Often the learning is intergenerational, but during family learning sessions it can be intergenerational for all or part of a session, whereas sometimes it might be for adults only.

The family learning programmes run by the social enterprise involved in the research are all run by experienced tutors who usually work with families and children in a wide variety of venues and settings. The courses range from Health Literacy to practical craft activities or Literacy, ESOL and Numeracy programmes. Before the pandemic the family learning courses would be in libraries, schools, community group venues, prisons or in outside settings. Where a course is run for the parents or carers specifically there will usually be arrangements made for a creche run by professional staff.

Research by NIACE found that 'parental engagement in family learning has a large and positive impact on children's learning, giving children greater confidence and self-belief, with measurable benefits to their literacy, language and numeracy skills' (NIACE, 2013, p. 7). Other research points to the positive benefits identified by parents, for both them and their children, ranging 'from practical and tangible gains (e.g. a certificate of achievement, improved vocabulary of the child) to emotional and psychological gains (e.g. rediscovery of 'old' self, increased confidence in themselves and their children)' (Brassett-Grundy, 2002, p. 41). The NIACE research also found that supporting children can actually help parents considered 'hardest to reach' to overcome practical, financial or dispositional barriers to learning.

Teaching approaches used in family learning contexts in this research have similarities to those described by González, Andrade, Civil and Moll (2001) working with low-income and minority students in Arizona. They describe families as "funds of knowledge" and view students as repositories of diverse knowledge bases which tutors build on to 'mathematize' household practices. They argue learning is developed in "zones of practice" where:

We can invite children into a world with a concrete motivating activity in which the everyday and spontaneous come into contact with the scientific and the schooled. The dichotomy of in-school and outside-school mathematics can be elided into a dialectical practice within which students' engagement with both the activity and the social context are foregrounded. (p. 128)

In this research the family learning sessions also seek to build on the outside-school knowledge parents and children already have as a starting point to develop a meaningful understanding of concepts identified in the school curriculum as useful in wider society. Family learning tutors often refer to themselves as “facilitators” to underline the strengths-based approaches that the family learning team use in sharing learning and supporting the identification and development of existing skills.

But Ashton (2013) also argues that family learning programs need to help parents develop ‘some knowledge of how to teach (pedagogy) as well as what to teach, to be able to support their children at home’ (p. 214). Research by Letzel, Pozasi and Schneider (2020) carried out during lockdown, supports this in that they found learning at home during lockdown was stressful for parents who were inexperienced in the range of demands of teaching, having little experience of the ‘pedagogical content for knowledge as well as an unfamiliarity of didactics’ (p. 166).

Research by Carpentieri, Fairfax-Cholmeley, Litster and Vorhaus (2011) into family learning programmes points to them needing to contribute to an even wider range of parents’ understandings including the cognitive, social and emotional development of the child.

Whalley (2007) contends professionals working with parents too often assume a deficit model, labelling some families ‘hard to reach’ or assuming they have no interest in their children’s learning. Both she and Morton (2010) argue for a closer involvement of parents in their child’s education where educators should listen to parents’ needs and ideas. The model promoted by the family learning team in this research seeks to build a closer, more collaborative relationship between the tutors and the parents of children involved in the online learning, hence this research.

While research into family learning is growing, Carpentieri et al, (2011) found it rarely explores further aspects of learning such as ‘family numeracy’ or the influence of technologies on the family learning experience. Even so, the research that has been undertaken identifies many perceived benefits to family learning which are shared between the child and the parent, even spreading to the wider community.

Learning Mathematics

Psychological research into mathematical cognitive processes by Whitebread and Bingham (2013) posit that a child’s understanding of their world develops dramatically in the first four years. They contend young children learn through observation and induction, that is detecting patterns in their experiences and constructing their conceptual knowledge of the world. These are part of what they call the development of a child’s executive functions that lead to habit formation in later life. They argue two- and three-year-olds are capable of understanding one-to–one correspondence and ordering, that is knowing numbers. Four-year-olds can be helped to make comparisons between characteristics such as heavier/lighter or bigger/smaller while between four and ten children can start to develop planning skills by discussing birthdays or measuring a number of sleeps until an event. They identify teachers and caregivers, such as parents, as influential in helping construct those concepts. In the UK, Ashton (2013) suggests family learning tutors should be aware of the mathematical content in both the pre-school Foundation stage curriculum and the adult numeracy curriculum. Taking into account the parents' learning needs as well as the child’s. She also suggests linking content to the strands of the UK national curriculum for 5–11-year-olds, which includes: using and applying mathematics, counting and understanding number, knowing and using number facts, calculating, understanding shape, measuring and handling data.

Research into younger children’s development of mathematical understanding and attitudes finds that both formal and informal family learning activities are important when developing mathematical skills, knowledge and attitudes. Skwarchuk, Sowinski, & LeFevre, (2014) research found:

Formal home numeracy practices (e.g., practicing simple sums) predicted children's symbolic number system knowledge, whereas reports of informal exposure to games with numerical content (measured indirectly through parents' knowledge of children's games) predicted children's non-symbolic arithmetic, as did numeracy attitudes (e.g., parents' enjoyment of numeracy). (p. 63)

This research points to the importance of families playing games together as well as 'doing school based learning'.

Grant (2009) argues that family learning can be an intergenerational experience when children learn from parents and parents learn from children, for example while undertaking Internet searches. He suggests parents might understand the mathematical subject content while children can explain how to utilise different digital platforms. However, this learning depends on the level of knowledge of children and parents in both technology and mathematics. As research by Civil, Díez-Palomar, Menéndez and Acosta-Iriqui (2008) posit mathematical interactions between parents and children are also influenced by previous learning experiences, cultural identities and age/generation. Their research points to a possible source of tension between children and parents when there is a 'gap' in how they learned a mathematical concept. Ashton (2013) further contends parents struggle to support their children if they do not have an up-to-date understanding of how mathematics is taught and cannot relate their knowledge to the current school curriculum.

However, research (Civil et al, 2008; Ashton, 2013) also suggests parents learning current school methods is helpful to their children, improves parents' confidence, and enables a dialogue with children about different mathematical methods. Research also shows helping their children to understand the school curriculum can be a real motivating factor for parents to return to learning and can be rewarding if parents can overcome previous negative experiences (Ashton, 2013; Kelly, 2019). In family learning programmes mathematical experiences can mean teachers, parents, and children are all learners.

On-line learning

Intergenerational learning online is still a relatively new phenomenon. Research undertaken during the first lockdown in Spain where learning dialogic literary groups transferred online, observed that some families found the experience online quite supportive and emotionally reassuring to be able to retain virtual friendships (Ruiz-Eugenio, Roca-Campos, León-Jiménez & Ramis-Salas, 2020). Research by Grant also argues that families learning with digital technologies can contribute to the narrowing of 'the achievement gap for children from disadvantaged backgrounds, while developing parental skills and strengthening intergenerational relations contributing to creating cohesive communities' (Grant, 2009, p. 2). However further research found transitioning to online learning during the Covid-19 pandemic resulted in school children from disadvantaged groups engaging less with learning thus exacerbating the learning gap in children from disadvantaged homes (Catalano, Torff & Anderson, 2021; Bonal & González (2020). Nevertheless, Barr argues that media now comprise a significant part of daily experiences for young children and that 'research demonstrates positive associations between joint media engagement of age-appropriate, well-designed media content and child outcomes and negative associations between 'technoference' [where technology interrupts interpersonal reactions] and child outcomes.' (Barr, 2019, p. 344) She advocates more research into the digital ecology of the family.

Borba and Villarreal (2005) suggest that using digital tools for online learning has been viewed historically as two distinct concepts whereby technology either 'serves' humans or dominates them. They posit that learning on and through computers influences the way humans understand things, so knowledge produced is a result of a collective process of humans and things. They argue that learning online is not only influenced by the technology but also influences the way we use the technology to think. Therefore, our understanding is an 'intershaping' relationship between humans and technology.

Englebrecht, Llinares and Borba (2020) further contend that the Covid-19 pandemic is encouraging more educators to use online learning thus developing a greater understanding of this relationship, creating what they now call knowledge that is developed collectively as 'humans-with-media'. They see humans-with-media affecting education, reasoning the internet is changing the way generations learn so that the traditional approach which is teacher driven, where the teacher is the source of knowledge and students passively absorb ideas (known as pushing knowledge) is being changed to a more student driven approach where students take control of the learning process—referred to as a pull process (Bassendowski & Petrucka, 2013). For example, art is a subject that traditionally utilises the pull approach to learning, requiring students to explore media and resources and bring those ideas back to merge with their own creative thinking to develop new art forms. Mathematics, on the other hand, traditionally lends itself to a push approach to developing knowledge where the teacher introduces the topics, methods and approaches used to solve problems in particular sequences. Online learning offers opportunities to change teaching approaches allowing students to select and transform information, enabling them to explore ideas that are interesting to them thus designing their own curricula. Bassendowski and Petrucka, (2013) argue learners can do this in an online environment that ideally supports collaborative discussion opportunities which they propose leads to more critical thinking, encouraging a more independent learner. However, utilising the new set of skills and knowledge that technology offers mathematics teachers may be contentious and require significant professional development, something that was not available during the first lockdown.

In her research into learning during lockdown in Wales, Lyakhova (2021) uses the term 'remote teaching' to describe learning not being undertaken face-to-face. She suggests remote learning describes a wide range of different models. These include 'blended learning', incorporating both online and face-to-face learning as well as 'flipped' classes, where topics are introduced to individuals through online videos, before a face-to-face discussion with other learners. Yorganci (2020) argues that introducing real life problems through short videos and quizzes before the class promotes discussion into understanding when the class meets together online. However, Yorganci's research (2020) was undertaken with older learners whereas Lyakhova (2020) points to the very limited amount of research into remote learning, especially its impact on younger learners.

When undertaking research into remote teaching Lyakhova (2020) also describes teachers as having to plan for a 'social, cognitive and teaching presence' (p. 18). This idea builds on research into online learning labelled a 'community of inquiry framework' by Garrison and Arbaugh (2007) which explores changes into each 'presence' when engaging with new groups online. Lyakhova (2020) argues parents of younger learners may have to take on a more 'social' role of mentor, encouraging children to concentrate, while the tutors focus on teaching the subject content. She suggests the younger the learner the more 'social' support they may need. Garrison and Arbaugh (2007) also point to the need for more research into changes in on-line teaching approaches, exploring possible changes in the balance between facilitation and direct instruction.

This literature review necessarily covers a range of ideas that represent an intersection of pedagogies of family learning online focusing on implications for mathematics teaching. The review does not claim to be comprehensive but brings together seemingly diverse concepts in order to explore the challenges and opportunities arising from using a family learning approach delivered through digital mediums to develop knowledge and skills in mathematics. Family learning is aimed at parents who are ready and interested in learning how to support their child's development. Parents have to give time to the learning activity while finding the physical space for children to learn and the digital devices required to do that learning as well as pay for connectivity. These pressures can be exacerbated if a household includes a number of children of different age groups which requires differentiation of that social support for teaching. These extra demands on parents' time and resources proved to be difficult for some, especially during the first lockdown.

The research also shows that as well as putting new expectations on parents and children, learning mathematics online also puts new demands on tutors who are expected to utilise and adapt to digital tools and platforms to promote learning. Developing children's mathematics skills and knowledge challenges tutors who have honed their teaching skills in the physical classroom to modify, or even revise completely, those approaches to adapt to a digital environment. The fact that the online world might often be better understood by the younger learners than their tutors also puts extra pressure on tutors hence has implications for their professional development. Being pushed or forced to undertake learning online places new demands on parents, children and tutors and requires further research to help understand the implications and develop support strategies for all involved.

Methodology

The research team used an action research approach to engage with adults who were participating in a range of online family learning courses provided through a small social enterprise in London during lockdown (March-April 2020). Action research is used widely in UK Education to encourage a reflective cycle in teaching, starting with identifying an issue, moving on to gathering information in order to interpret the data and then analysing the evidence before finally evaluating the results to recommend improvements (McAteer, 2013).

In this research the aim is to listen to the ideas of parents involved in the family learning programmes which were developed for face-to-face teaching but moved online at short notice. The tutors were keen to listen to the experiences of the parents and consider how the online learning environment might be improved. We wanted the words of interviewees to lead our analysis and grounded our theory in words of the interviewees which enabled us to 'follow leads that emerged out of the data collected' (Charmaz, 2006, p 14) while systematically building theory or ideas from data.

Research sample

The research sample of participants contained 20 parents or carers and three family learning tutors. The carer group consisted of 19 females and one male. Eight people in the sample classified themselves as Black African, four as British Bangladeshi, three as other white, two as other Asian, one as white British, one as Black Caribbean and one did not classify themselves. Six of the adults in the sample were between 16 years and 25 while 14 were aged between 25 and 36 years. A total of 48 children are connected to the adults involved in the research, a majority at primary school or nursery age, 34 were below 11 years old 25 of these were below six years old. All of the participants live in South London, in the UK.

The three family learning tutors were all involved in teaching online during the first lockdown. They are part of a group of five family learning tutors involved in teaching through the social enterprise, all female with a broad range of skills and teaching experience. In the sample, one of them describes herself as Black Caribbean, the other two as White British. Most of the tutors live and work in London boroughs although some are also involved in planning courses that would be run elsewhere in the country. All tutors have the Initial Teacher Training qualifications for adults and some are also qualified to teach children.

Contacting participants for the research

Invitations to join in this research were sent out through a variety of networks to ensure a sample size that was authentic and credible. Authentic in the sense that it is important to recognise various viewpoints that stakeholders hold within different social worlds, and by so doing, aims to arrive at better understandings (Bryman, 2012, p. 392). Authentic also in that this research seeks to give a voice to a perspective less often heard, that of parents and carers experiencing family learning. Credible in the sense that we were able to carry out in-depth interviews with twenty carers and three tutors to gain multiple perspectives that 'reflect participants', researchers' and readers' experiences with a phenomenon' (Corbin and Strauss, 2008, p. 302).

Lefever, Dal and Matthíasdóttir (2006) found online requests for data were variable and often had low response rates. In this research invitations varied in that they were given to parents who were participating in the online courses in children's schools, children's centres and libraries. They were also given through word of mouth from learners on a range of programmes and through social media. Their engagement in the interviews was entirely voluntary. The list of courses, their purposes and timings are listed in Appendix 3, along with the number of participants from each programme who participated in this research.

The participants in the interviews were identified and recruited by the online family learning tutors. This method of sampling was purposive (Bryman, 2012, p. 422) in that adults interviewed fulfilled a particular set of criteria, essentially that they attended the new online family learning courses, which started during lockdown in March 2020. Two of the family learning teachers also joined the research team and could be seen as 'gatekeepers' to accessing participants. Atkinson (1986) warns against a reliance on 'gatekeepers' for the research data as they could be engaged in 'impression management' where tutors could be hoping to choose interviewees who would give a good impression of the work they did and the results they achieved. However, in this research the questions focused on more general experiences of online learning and learning mathematics. The research team also included two interviewers unknown to the respondents, while others completed online questionnaires hence these participants were less open to any management of data or bias. The identification of specific courses families attended is used to provide background context to the main purpose of the research. See Appendix 1 for the full set of questions.

The interview requests and outcomes were as follows: 11 were asked verbally during an online course or workshop, all of whom agreed to participate but only seven resulted in interviews, with one online completion. There were 25 requests to participate made by email, 12 agreed but again only seven resulted in interviews, two members preferring to complete the questionnaire online. A request was sent out to all of the people who attended or interacted through Facebook interventions but this resulted in only three completions of the online questionnaire.

Collecting the data

The research team developed the questions for the interviews and one team member put the questionnaire online. At the beginning of each interview and written at the top of the online questionnaires was a statement explaining to the interviewee the purpose of the research and that we were following ethical guidelines on data collection (see Appendix 2).

No data was collected to indicate how long self-completed online questionnaires took but the semi-structured interviews took 30 to 45 minutes. Each person who participated in the research answered the same set of questions and was assured anonymity. The technology behind the online set of questions also enabled the team to collect and store the data anonymously, identified only by a time and date.

As we were undertaking action research, we have also included reflective pieces of narrative by three of the five family learning tutors who taught online and volunteered to be part of the research. This enabled us to perform a type of triangulation of the data. In action research social phenomena can be cross checked from different sources and used to compare and contrast different perspectives as well as identifying pointers for future research (McAteer, 2013).

The research team

The research team consisted of two family learning tutors, one tutor specialising in dyslexia and literacy and another tutor who specialises in numeracy and mathematics. All the research team were involved with the collection of data and analysis. The data collected was qualitative and the analysis of the data was a collaborative endeavour. The research team members worked together to identify themes in the data by looking for repetitions in responses, similarities and differences in topics were identified across the sources as well as metaphors and analogies used by participants to express their thoughts (Bryman, 2012, p. 580). This process of research and analysis helps to break teacher isolation fostering a culture that promotes learning for all (Sagor, 2010).

Findings and Analysis

In this section, we describe the findings from the research in relation to online learning during the March to April lockdown in the UK. We group the data into sections, initially focusing on the subject area of mathematics, then exploring the adults' general learning experiences, followed by tutors' reflections on their experiences. This research has contributed to the teachers developing online family learning experiences that contributes to the development of a joint curriculum with parents.

Focusing on mathematics

Half of the participants in the sample (10 out of 20) had accessed at least one mathematics website during lockdown. The websites used were from a wide range focusing on mathematics for younger learners, which included school sites. Two of the families paid for instructions through commercial websites, one explaining that the approach "…is more formal learning, set papers are given to work on every day for a week and then they provide zoom sessions to go through the work." One parent chose particular websites because "I find these websites very good and my daughter likes them" while another said the website was "great because the activities are not too long, just right and easy enough for the children to do without support." Others had difficulty finding what they were looking for online, as one explained:

> Numeracy was what worried me the most about teaching my children at home. It was difficult to find free resources online. There were no short/sharp courses to upskill my maths but luckily my brother has stepped in.

This quote indicates a need for some parents to have further support and guidance when trying to assist in their children's mathematics development. As another explained:

> Sometimes he [her son] doesn't understand the maths so I have to use the methods they use [at school or online] to help explain it to him.

Another spoke about being challenged with explaining more complex mathematical concepts, saying she was happy with addition and subtraction but did not know how to introduce multiplication. All these statements indicate adults are aware of the need to develop their own knowledge and understanding of mathematics in order to support their children. The notion of mathematics being a subject that causes some anxieties for parents was indicated in the first long quotation above but is explained more explicitly by another who said:

> I don't like online learning and mathematics is always a bit painful for us but BBC Bitesize maths games are good.

This statement is interesting, although the adult expressed really negative emotions towards mathematics, they found BBC Bitesize games 'good', indicating the possibility of ways of engaging learners to overcome their anxieties when motivated to learn to help their children.

These few quotations provide further data supporting research indicating that parents can overcome their maths anxiety when motivated by their children (Kelly, 2019) and become learners themselves (González, Andrade, Civil & Moll, 2001). What might be viewed as more concerning is that only half of those interviewed accessed any mathematics resources online. The variety of responses, even within this small group, indicates the need for further research to try to develop meaningful support for families to extend their mathematics skills together.

Parents', carers' and children's experiences

A majority of parents and carers (17) responded positively to a question about the usefulness of learning online, while 15 spoke about learning online helping both their own and their children's confidence grow. This indicates an appreciation by parents of both themselves and their children as learners. A small but significant group (4) found the loss of physical contact with adults in the family learning context (i.e. the community of learners) more difficult.

Five parents stated they enjoyed learning with their children and developing an understanding of how to support children's learning. For example, one said she liked "spending more time with my children and I learnt from them sometimes.'' The families enjoyed sessions that were interactive finding them motivational and three stated they enjoyed the 'warm up' activities at the beginning of the sessions. As one said, "Early in the morning you don't feel like doing much but felt ready afterwards." While another indicated she liked it when the tutor had an activity planned for the time spent waiting for others to join the class. This last notion indicates a useful approach for planning future sessions.

Participants also valued the regularity of sessions during lockdown, as one explained it "made me realise that for my children a zoom session with their teacher at least once a week would be really good." Another spoke about how the family "look[ed] forward to the next [session]," while another described the sessions as a much needed "break from the monotony."

At least five participants highlighted the 'teaching role' of online tutors as important during these sessions, ensuring a lead and purpose to the sessions. Teachers influenced children's behaviour by giving clear guidance on activities and resources. As one parent observed "….my daughter will do more for the online teacher than for me, as they want to impress her." Five spoke about how having to manage their children's behaviour and how it was affecting their personal learning. As one explained "If I want to learn something, they distract me" and another said it was difficult "…keeping my children quiet." Yet another said it is "…difficult to keep them engaged", while another pointed to the "…need to be around constantly to support children with learning and technical difficulties." All these quotes point to the need for parents to develop their own skills in providing 'social' support and motivation for their children's learning. In the family learning context tutors aim to focus on parents' understanding of concepts and enable those parents to support their child's learning.

When considering how to improve both adults and children's online learning experiences participants asked for more 'talk time' in future sessions; 10 wanted more time to share experiences with other parents, eight wanted more 'talk time' with the teacher, two others suggested it would be good to give the children more 'talk time', while two others suggested having longer sessions to include breaks for talking. This research was undertaken during the first lockdown when there was little movement outside the home. This need to talk can be linked to the 'social' side of learning with peers, i.e. the learning community, which was missing from the online experience where short informal chats are not possible.

Further challenges to the 'social' aspect of learning were identified by six people who spoke about the trials of using technology and Zoom when interacting with others. Some participants reported they found the medium awkward, one explained "You can't read people's feelings and emotions." Another described the problem as "the conversation in class can't flow so easily online, it can get noisy if people aren't muted during class." Psychologists talk about the lack of ability to receive cues through zoom which leads to participants in meetings 'not feeling "in tune" with speakers, so their interaction is not smooth.' (Bailenson, 2021, p. 3)

One participant who was unable to access a computer pointed to an important limitation when learning online, when she was forced to attend the sessions using only a smartphone because “It was much harder to follow the course on a small screen and take notes at the same time.”

Overall, the parents and carers responses indicated they valued the sessions with the tutors online during lockdown, suggesting approaches and activities that could be further developed in future online family learning courses. However, parents also pointed to some of the challenges undertaking family learning through technology, the need for more ‘talk time’ suggests the limitations of the social experience for adults and children. The difficulty with reading peoples’ emotions also leads to adults having to learn new ways to interact with other parents and their children. All of these issues were further exacerbated when there was restricted access to appropriate digital devices.

The responses also highlighted the need for parents to recognise and take on a social role to support the teacher when their children were learning online. Parents themselves spoke about having to learn more about ‘how’ their children learn, in order to support the child’s understanding of particular subject concepts, which accords with the family learning ethos. They also spoke about the need to motivate their children to attend sessions as well as encouraging them to stay focused, again reinforcing the important social role that parents play in online learning.

Tutors’ reflections

As part of the evolving collaborative approach to developing online family learning courses, three of the all-female family learning tutors who engaged in the online learning experiences volunteered their reflections. The tutors found moving to the remote delivery of courses and workshops was “fun but also challenging”. They reported being inspired when observing the motivation and determination of parents’ willingness to adapt to online learning.

However, the tutors were very aware that teaching online during lockdown meant that while some participants had entry to courses who might not have had access previously, it also meant that learners with no access to the internet or digital devices were unable to engage with the family learning programmes online.

Tutors reported finding many differences between teaching face-to-face and online. They spoke about the need to engage and understand the technology and how it changed their ability to communicate with people and children, in the first instance the ability to develop an immediate rapport is more challenging. In practical terms, it is harder to work on a one–to-one basis with learners, to help them develop their skills.

One tutor stated, “If a learner did not want to be seen and the screen is turned off, on Zoom, then it is impossible to see facial expressions and limited levels of engagement.” This is one of the barriers created by digital platforms such as Zoom. Tutors also spoke about finding it impossible to have spontaneous private conversations with students unlike the classroom. They found it difficult to take a learner to one side to discuss a particular issue, even with breakout rooms. If this was needed the tutor had to arrange another meeting outside the session. Because only one person can speak at a time online, discussing a topic in pairs or small groups is more difficult unless breakout rooms are used, even then some tutors found that difficult to manage as they can “only be in one room at a time”. The tutors found this process led to less interaction between the learners and less class discussion in general, tending to make the pedagogic style more teacher focused.

All tutors said they found online teaching more difficult than face-to-face and had to find inventive ways to get to know individual learning needs within the group and to build trust. Differentiating between individual learners' requirements and providing additional support to meet their needs was problematic as it tended to disturb the rest of the class and disrupt the group dynamics. This became very apparent when intermittent internet connections resulted in some individuals missing part of the session while they re-established the link. Providing individualised catch-up for those who missed sections of learning, through no fault of their own, became problematic.

A general reflection was how teaching online is more tiring than face-to-face. One suggestion was that sitting in front of a screen was difficult for those who like to move around when teaching. Psychological research points to “Zoom Fatigue” being a product of unnaturally long eye contact with strangers and the lack of non-verbal behavioural clues normally picked up when working in groups (Bailenson, 2021).

Completing forms for administrative purposes also had its challenges as many participants did not have printers or scanners. In response tutors tried to use Google forms, which were easier for the learners to complete but there tended to be fewer completions. There may also be privacy issues related to putting data into online forms.

In this first lockdown, many tutors rose to the challenge and moved their classes online but the findings suggest they had little available professional support to develop their own online teaching skills. Their willingness to try to sustain a link with adults and their families to continue learning during lockdown was commendable. Inevitably, the limitations on the availability of technological devices and connections took their toll for learners and tutors. The novelty of adapting to learning through technology put limitations on the tutors’ ability to manage the social dynamics of the group, restricting the more personalised and differentiated encouragement of adult learners and their children enabled in a classroom.

Discussion

Unsurprisingly the use of technology in family learning programmes, during a national lockdown, was a mixed experience for families and tutors. The research sample was restricted to only those people who had access to digital devices and the Internet. The 20 learner respondents offered an insight into often unheard voices and gives the research an important perspective, even if the sample is relatively small

Access to digital devices and connectivity was an issue for tutors and participants. Intermittent access to the internet caused disruption while tutors and learners may have had to learn how to use new video-conferencing platforms and apps which also posed their own challenges, as participants had to get used to different protocols and approaches to teaching and learning necessary to have successful online learning experiences. Nevertheless, of those learners who could connect, the majority enjoyed the interaction and the chance to engage with others, which was important for their wellbeing.

Learning online and supporting families in remote locations meant family learning tutors had to adapt their teaching and learning approaches. Tutors had to manage technical aspects of new software while trying to develop parents’ ability to support and motivate their children’s learning. All of this while also assessing adults and children’s learning needs. Tutors wrote about their experiences of working with families online as being varied, pointing to difficulties of “reading” how individuals found the sessions while using Zoom. One tutor reported she was able to assess participants’ learning needs because the classes were smaller than usual, however all tutors reported the difficulty of trying to work one-to-one with parents. Hence, the Covid-19 pandemic is changing teaching approaches ‘pushing’ (Morton, 2010), participants and tutors online to develop as ‘humans-in-media’ (Englebrecht, Llinares & Borba, 2020). Recognition of the demands of this new development and its impact on teaching and learning is not yet fully appreciated.

As described, mathematics traditionally lends itself to a ‘pushing knowledge’ model, online family learning lends itself more easily towards a ‘pulling knowledge’ model (Bassendowski & Petrucka, 2013). For example, tutors can encourage parents to use real objects from their homes, such as identifying balls as spheres, to help engage younger learners. Tutors teaching family learning mathematics also have a real opportunity to develop creative and collaborative new ways of learning mathematics, for example, signposting parents to useful websites which they can 'trust' or exploring the potential of those websites identified by parents. Investigating websites and the different ways they try to teach mathematics could also provide opportunities to explain the reasons why methods exemplified on some can support or even confuse their children, thus leading to parents' deeper understanding of how knowledge is developed.

However, in this research the tutors wrote about finding the technology created less social interaction between the learners and less class discussion in general, tending to make their pedagogic style more teacher focused. This is a point of interest as it seems to contradict the research that online learning offers opportunities to 'pull' knowledge from learners as opposed to 'pushing' knowledge into them. However, it further accords with findings by Garrison and Arbaugh (2007) suggesting a possible change in the balance between facilitation and direct instruction that could be explored in further research.

While children are growing up in a technological world their parents and grandparents are learning for the first time how to adapt to the online world. The need for parents to play a supportive social role in their children's learning experience is to be expected but research suggests this role may change as children grow older. A key aspect of good family learning is that parents can support their children's learning without being dependent on a teacher/tutor. However, when learning mathematics, especially as children age or parents are less confident with concepts, parental roles may change from supporting teachers in early years to focusing more on motivating children to learn while parents themselves become learners. Indeed, expectations of both parents and tutors about their roles may change throughout online learning sessions, which would contribute to the challenges faced during learning online and points to the need for further research.

Conclusions

The lockdown experience thrust upon UK society created many problems but also pushed all generations to develop their online presence. This research explores the experiences of family learning tutors and parents who rose to the challenge to develop online learning and identifies the important benefits as well as challenge experienced by a group of mostly minority background women learners (who are less often researched but are involved in family learning). Consequently, this article explores an interesting intersection of three pedagogies: family learning, online learning and the development of mathematical understanding.

The research identifies a need for further investigation into the new demands on teaching and learning through technology, including ways to overcome the social and psychological barriers experienced by both learners and tutors in family learning contexts. Additionally, the limitations and barriers in digital access are exposed. Nevertheless, the significant part that media now plays in children's daily lives can offer opportunities for collaborative intergenerational online learning for families as well as possibilities for change in mathematics teaching and learning. This highlights the growing need for research into the digital ecology of family learning, especially in mathematics education.

The findings in this small-scale research project underlines the need for further research into opportunities that technology offers to develop new approaches to learning online both in the family learning approach and the development of mathematics skills and knowledge. However, it also points to the need to support both tutors and parents to develop the skills needed for successful online interactions. Such research would inform teacher training courses in new pedagogic approaches essential to the successful development of online learning strategies for the future.

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Appendix 1. Family Learning Online Survey Questions

1.1 What do you use for online learning? (desktop, computer, phone, tablet, laptop)

1.2 Did you have any difficulties accessing the courses?

1.21 If you had difficulties, what were they?

1.3 Do you have a printer?

1.4 What age range are you in?

1.5 What age/s of child/ren are you supporting?

1.6 Do any of your children have any special needs or disabilities?

1.7 Any additional comments?

2.1 What was the most useful thing you found learning online? (or what did you like most)

2.2 What was the most challenging thing you found about learning online? (or what did you like least?)

2.3 Is there anything that could have made the experience better for you or your child/ren?

2.4 Any additional comments?

3.1 Which LU activity did you attend? (tick all that apply)

3.2 What made you choose LU online activities?

3.3 Which online activity / activities did you like the most or find the most helpful?

3.4 Which were the least helpful?

3.5 Do you think you or your child/dren have developed confidence through online learning?

3.6 What other courses would you like? (e.g, art, maths, healthy living, history, science ... other...)

3.7 Do you think the contact with LU has been useful for you emotionally or helped your wellbeing during lockdown?

3.8 Any additional comments?

4.1 Have you done any other online activities with your child/dren during lockdown, and if so, what?

4.2 What do you like about online learning courses with your child/ren?

4.3 What would improve your online experience? (e.g. more talking time with other parents, more talking time with the teacher...other ...)

4.4 Have you ever used free maths materials online?

4.4.1 If yes, which materials / websites?

4.4.2 If no, any particular reason?

4.5 Any additional comments?

5. Is there anything else you would like to say about supporting children's or your own learning during lockdown?

Appendix 2 Ethical statement

Interviewees had the following statement read to them before undertaking interviews or available to read prior to undertaking the online questionnaire.

Thank you very much for agreeing to take part in this research interview about your experiences of learning online with your children during lockdown. The research is to help Learning Unlimited tutors to understand better what you thought were helpful online activities and what was less helpful. This will then be shared with colleagues to help improve the content of our courses, from your point of view.

All of the information you give me will be anonymised and should you decide to withdraw your answers you will be able to do this up until August 30th and your answers will be removed from the data. We will not share any of your personal data with any other organization.

All of the data collected will be held anonymously and kept under secure conditions. The data will be destroyed at the end of the research project.

Are you still happy to go ahead with the interview?

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Building Thinking Classrooms Online: From Practice to Theory and Back Again

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Abstract

In the COVID-19 era of adapting to pandemic lockdown protocol, teaching practices have become more negotiable and less tethered to the familiar and institutionally normative practices found in educational settings around the world. With a shift to online teaching, many practices are being adapted from face-to-face settings and being imported into online settings. However, this sort of adaptation is by no means trivial, and a direct transfer of practices may not necessarily be effective or plausible. While adaptation is undeniably necessary, a theory *for* teaching can offer guideposts around which adaptation may occur. Over many years of empirical investigation into how to enhance the synergy and capacity of students' thinking in face-to-face mathematics classrooms through systematically bypassing institutionally normative practices, the *Building Thinking Classrooms* framework offers a basis for one such theory. While this framework is used in many different contexts, one of these is in the education and professional development of mathematics teachers in tertiary and professional settings. However, with COVID-19 protocols in place, the tightly woven face-to-face practices of this framework had to evolve and be adapted. In this article, we discuss and exemplify how we drew from these face-to-face practices a set of principles, which served as guideposts for designing adaptations for engaging adult learners in mathematical tasks in a fully online setting. In our analysis, we consider not only the adaptations for online teaching we made, but the process of adaptation through a theory *for* teaching we used in designing effective and intentional learning settings for adults experiencing mathematics.

Key words: mathematics, online, teaching practice, teacher education, theory for teaching building thinking classrooms

Introduction

Adult learners return to the study of mathematics for a variety of reasons (e.g., to fulfill economic needs, for personal fulfillment, etc.) and the learning contexts in which they do so vary widely (e.g., parent education, financial literacy, workplace and vocational education, adult basic education, pre-service and in-service teacher education, etc.) (Safford-Ramus, Misra, & Maguire, 2016). Regardless of context, adult learners face various boundaries and barriers towards learning mathematics based on their past learning experiences and life situations (FitzSimons, 2019). Their personal responsibilities and life pressures make them aware of why they are learning something and how they can apply it in their lives (Knowles, Holton, & Swanson, 1998). In turn, they desire an active role in decisions and discourse in a learning environment (FitzSimons & Godden, 2000). Adults have also been "positioned by practices of curriculum (Popkewitz, 1997), pedagogies and psychologies about mathematical reasoning and learning (Popkewitz, 1988; Walkerdine, 1994), and textbooks (Dowling, 1998), [and] these practices are not neutral but reflect larger economic, cultural and political considerations" (FitzSimons & Godden, 2000, p. 15). The multiple and overlapping subjectivities adult learners carry are called up by a range of classroom practices and are further shaped by new classroom practices they encounter. As such, teaching practices used with adults who are learning mathematics in post-compulsory settings require careful attention about how they shape their experiences of doing mathematics, and in consequence, of thinking mathematically. It is thus important for practitioners to be reflective and cognizant of their own practices and how they acknowledge the adult learner's needs for engaging in the thinking process. Moreover, it is important for practitioners to consider how practices can be adapted when contexts change since every teaching context provides novel challenges related to engaging adults in mathematical thinking.

While there are many approaches to teaching adults mathematics, our interest in this paper is to examine our own teaching practices used with adults learning mathematics in tertiary pre-service teacher education and teacher professional development settings in which we adopted a Building Thinking Classrooms (BTC) (Liljedahl, 2016, 2020) model of instruction. In particular, we examine how we shifted our teaching practices from those that were appropriate in our face-to-face settings, to those we used in fully online settings with these populations in response to the limitations created by the COVID-19 pandemic. While we do not extend our discussion to the experiences of our adult learners in this paper, we choose instead to focus on our interpretation of adapting our teaching practices to meet the needs of our learners in a new context. By investigating our own practice, we are serving the global aims of improving the learning experience for adults learning mathematics in our context of tertiary and professional education. We are also revealing a viable approach to adaptation of teaching practice.

To this end, we first visit the roots of where our face-to-face practices emerge from by reviewing how the BTC model of instruction arose, what it is, and how we used it in our adult settings. We then reveal how we approached designing adaptations of the BTC model for the fully online environment and showcase how we implemented some of these adaptations

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The Emergence of Practice

The teaching of adults in tertiary and professional settings can look very much the same the world over. For the most part, it follows a model of demonstration and reproduction – what is often called an I do—we do—you do approach to teaching. To understand why this is, we first take a brief look at the origins of public education and consider where these normative practices arose from.

Looking back at when the first industrial revolution came to a close, countries around the world at this time realized that if they wanted to continue to grow their economies, they would need to educate their citizenry. Out of this realization was born the concept of public education (Katz, 1987) and with it the institution of school, which was constructed to create conformity and compliance. To achieve this, public education was built on a foundation of the three institutions that were, at the time, seen as successful (Egan, 2002).

1. The *church*, which already had a mandate to educate the masses and from which the early designs of classrooms were drawn.
2. The *factory*, from which we learned the principles of mass production.
3. The *prison*, where had learned how to manage and move large numbers of people.

Together, the influences of these three institutions shaped what the classroom looked like, and, in turn, what teaching looked like at the dawn of public education. It was at this time that we saw the emergence of a pedagogical model that we now call *I do—we do—you do*. This model capitalized on the efficiencies of the factory while maintaining the control of prisons, and it looked like church, with the teacher at the front and all the students facing forward.

And through the process of *cultural reproduction* (Bourdieu & Passeron, 1990), classrooms of today, and the teaching that takes place inside them, still look very much the same. These norms transcend the classroom (Cobb, Wood, & Yackel, 1991; Yackel & Cobb, 1996) and have woven themselves into the very fabric of the institution of school - forming what can only be referred to as *institutional norms* (Liu & Liljedahl, 2012). But these norms transcend K-12 (primary and secondary) education and have infused themselves into what it means to teach in general – at all levels from primary to tertiary and for all audiences from children to adults. This is not to say that education has not changed over the course of the last 150 years. Curricula have evolved, there have been efforts to create access and equity in education, and the role of technology has vastly altered what is possible in (and out of) the instructional setting. The desks have evolved from church pews to desks to tables, and we have gone from blackboards to greenboards to whiteboards to smartboards. But much of what happens in K-12, tertiary, and professional development settings today is not too dissimilar to what happened in these settings a century ago. That is, although there has been great evolution of *what* is taught in the last 150 years, the institutional norms that were laid down at the dawn of public education still dictate much of *how* teaching looks in tertiary and professional settings today. Learners are still sitting, and instructors are still standing. Instructors are still writing on boards and learners are still writing in notebooks. And instructors are still following the *I do—we do—you do* pedagogical routine.

In our efforts at designing effective and intentional learning spaces with adults learning mathematics in our tertiary and professional settings, we asked: How do we change this? How do we break the cycle of cultural reproduction to change the experiences of our adult learners? One of the ways we have achieved this is by drawing on the research of Liljedahl (2016, 2020) on how to build thinking classrooms. This research offers a set of teaching practices developed systematically out of challenging institutionally normative practices. Although it was enacted in the K-12 setting, we have found numerous points of connection with the world of adult education and have been able to transfer he ideas seamlessly into our adult education settings. Since our face-to-face teaching practice is based on this research, we first discuss its highlights.

Building Thinking Classrooms

In visits to 40 different K-12 mathematics classrooms in 40 different schools, Liljedahl (2016, 2020) found that in all cases, the lesson began with some form of teacher demonstration (I do), followed by student replication either individually or in groups (you do), which in turn was followed by some form of consolidation (we do). Although the details of how this looked, the amount of time apportioned for each activity, the degree to which students worked in groups, and the degree to which technology and manipulatives were incorporated varied, what did not change was a general adherence to this routine. Liljedahl (2016, 2020) further observed that in a typical lesson, there was very little opportunity, and even less need, for students to do much thinking. Closer examination of this observation (Liljedahl, 2020, Liljedahl & Allan, 2013) revealed that in a typical mathematics lesson only about 20% of the students did any real thinking and, even then, only for about 20% of the lesson. Instead, students relied on a slate of behaviors that included slacking, stalling, faking, and mimicking to slide through the lesson without thinking. Liljedahl (2016, 2020) attributed this to the aforementioned institutional norms that not only dictate many of the activities of teaching, but also the activities of learning.

Liljedahl (2016, 2020) posited that for this reality to change – in order to get more students thinking and thinking for longer – a radical departure from the institutional norms would be needed. And thus was born the *Building Thinking Classrooms* (BTC) project which, for over 15 years, sought to empirically emerge and test pedagogical practices that not only afford opportunities to think, but that necessitate thinking and increase thinking in the classroom. This work was organized around the 14 general categories of practice that all teachers adhere to in some shape or form.

1. What types of tasks we use.
2. How we form collaborative groups.
3. Where students work.
4. How we arrange the furniture.
5. How we answer questions.
6. When, where, and how we give tasks.
7. What homework looks like.

8. How we foster student autonomy.
9. How we use hints and extensions to further understanding.
10. How we consolidate a lesson.
11. How students take notes.
12. What we choose to evaluate.
13. How we use formative assessment.
14. How we grade.

Each of these general practices served as a variable in the research, which involved more than 400 K-12 teachers implementing thousands of two-week micro-experiments, each of which sought to measure the degree to which a specific practice impacted the amount of thinking observed. More details about methodologies involved and results can be found in Liljedahl (2016, 2020).

Emerging out of this research are 14 teaching practices, one for each general practice, that have been proven to produce more thinking in the classroom than the institutionally normative practices they sought to replace as well as more thinking than any of the other hundreds of practices experimented with (Liljedahl, 2020). These practices are described briefly below.

1. *The types of tasks we use*: Lessons should begin with good problem-solving tasks. At the beginning, highly engaging, non-curricular tasks are used, but after a period of time, they can be gradually replaced with curricular problem-solving tasks.
2. *How collaborative groups are formed*: At the beginning of every class, a visibly random method should be used to create groups of three to will work together that day.
3. *Where students work*: Groups should stand and work on vertical non-permanent surfaces (VNPS) such as whiteboards, blackboards, or windows, making work visible to the teacher and other groups.
4. *How we arrange the furniture*: The classroom should be de-fronted with desks placed in a random configuration around the room (but away from the walls) and the teacher addresses the class from a variety of locations within the room.
5. *How we answer questions*: Teachers should only answer the third of three types of questions that students ask: (1) proximity questions – which are questions asked merely because the teacher is close; (2) stop thinking questions – which are questions that aim to cease thinking e.g., “is this right” or “will this be on the test”; and (3) keep thinking questions – which are questions that get them back to work.
6. *When, where, and how we give tasks*: The teacher should give tasks verbally (as much as possible) at the beginning of the session from a non-central location in the room after gathering students around them. If there are data, diagrams, or long expressions in the task then these are written or projected on a wall, but the instructions pertaining to the activity of the task should be given verbally.
7. *What homework looks like*: Rather than assigning homework or practice questions, students should be assigned 4-6 questions for them to check their understanding. Students should have the freedom to work on these in self-selected groups or on their own, and on the vertical non-permanent surfaces or in their desks, and should be for self-evaluation and not marked or checked.

8. *How we foster student autonomy*: Students should interact with other groups extensively, both for the purposes of extending their work and getting help. As much as possible, the teacher should encourage this interaction by directing students towards other groups.
9. *How we use hints and extensions to further student understanding*: The teacher should maintain student engagement through a judicious and timely use of hints and extensions to maintain a balance between the challenge of the current task and the abilities of the students working on it.
10. *How we consolidate a lesson*: When every group has passed a minimum threshold, the teacher should pull the students together to debrief what they have been doing. This debrief should begin at a level that every student in the room can participate in.
11. *How students take notes*: Notes should consist of meaningful notes written by students to their future selves. Students should have autonomy of what goes in these notes and how they are formatted, and the notes should be based on work that has already taken place.
12. *What we choose to evaluate*: Summative assessment should honour the activities of a thinking classroom (evaluate what you value) through a focus on the processes of learning more so than the products. It should not in any way have a focus on ranking.
13. *How we use formative assessment*: Formative assessment should be focused primarily on informing students about where they are and where they are going in their learning. This requires, by necessity, several different activities from observation to *check your understanding* questions to unmarked quizzes where the teacher helps students to decode their demonstrated understandings.
14. *How we grade*: Reporting out of students' performance should be based on the analysis of the data, rather than the counting of points, collected for each student within a reporting cycle. These data need to be analysed on a differentiated basis and be focused on discerning the learning that a student has demonstrated.

Although this set of BTC practices emerged from research in the K-12 setting, they have been used effectively in adult tertiary and professional learning settings such as in adult basic education courses (Larsen, 2018a; 2018b), teacher education courses (Mellone, Pacelli, & Liljedahl, 2021), and in teacher professional development settings (Andrà, Rouleau, Liljedahl, & Di Martino, 2019; Liljedahl, Andrà, Di Martino, & Rouleau, 2015; Rouleau & Liljedahl, 2016; Rouleau, Ruiz, Reyes, & Liljedahl, 2019). While significant efforts have been made in adult mathematics education settings to support some of the BTC practices such as prioritizing collaboration (e.g., Gibney, 2014; Oughton, 2009), fostering autonomy (e.g., Larsen, 2015; Yen & Liu, 2009), and involving formative assessment (e.g., Looney, 2007), the BTC framework offers a complete set of practices that work in conjunction with each other to ensure a context that is rich in student thinking. In many cases, the implementation of this set of practices makes a radical departure from the prevailing norms of the context, particularly in larger institutional settings. In other cases, such as in more informal or progressive contexts, less so. Regardless, this set of practices marks a significant evolution of teaching from the institutionally normative practices that permeate and have permeated education for the last 150 years and offer us new ways to enact teaching in the face-to-face classroom

Disruptions of COVID-19: From Practice to Theory

But then in early 2020, COVID-19 hit. Suddenly, the practices that had taken 15 years to emerge out of face-to-face classrooms no longer seemed relevant to the online setting that COVID-19 necessitated. Digital modalities replaced every bit of classroom practice. Group work became more challenging to arrange, hands-on activities became next to impossible, and communication was constrained. Every aspect of practice was now challenged by new contextual obstacles. Teachers around the world were sharing this struggle and some found it easier than others to tailor their face-to-face practices into the constraints set by online learning tools.

We, the authors, were regularly implementing BTC practices in adult tertiary and professional learning settings prior to the COVID-19 pandemic. When the pandemic forced us to move into a fully online learning setting, we were faced with adapting practices developed in face-to-face environments to fit into an online environment unfamiliar to us or our students. Rather than adapting BTC practices to the online setting in a random fashion, we decided to conduct a thematic analysis of the BTC practices themselves to identify the core guiding principles behind each practice. Such an analysis would help identify aspects of the BTC practices that were important to retain as we adapted to teaching in a novel context (i.e., the fully online learning setting) while maintaining the integrity of their aims of prioritizing learner thinking over *I do—we do—you do*.

In essence, we aimed to extract from the empirically deduced BTC framework of practice a theory *for* teaching. We did so deliberately and with full knowledge of the controversy this can invoke. We understand that in mathematics education the idea of a theory is reserved for constructs that are empirically, philosophically, and theoretically deduced explanations for natural phenomena, such as learning, and not intentional actions, such as teaching.

While theory provides us with lenses for analysing learning (Lerman, 2001), the big theories do not seem to offer clear insights to teaching and ways in which teaching addresses the promotion of mathematics learning. (Jaworski, 2006, p. 188).

And, because of this, a theory *of* teaching is not possible.

Theories help us to analyse, or explain, but they do not provide recipes for action; rarely do they provide direct guidance for practice. We can analyse or explain mathematics learning from theoretical perspectives, but it is naive to assume or postulate theoretically derivative models or methods through which learning is supposed to happen. Research shows that the sociocultural settings in which learning and teaching take place are too complex for such behavioural association (Jaworski, 2006, p. 188).

However, we were not trying to create a theory *of* teaching. Rather, we were trying to extract from the BTC framework a theory *for* teaching–a theory that does not *explain* teaching, but that *guides* teaching. There is a distinction between theory *of* and theory *for*. In what follows, we show how we developed this theory *for* teaching as a set of *principles of practice* and then articulate how we used these results to adapt BTC practices for the online setting with adult learners, such as those made necessary by the COVID-19 pandemic.

Method

Towards developing a set of principles of practice from a series of practices, we chose to apply a thematic analysis. Thematic analysis (Braun & Clarke, 2012) is a systematic way of "identifying, organizing, and offering insight into patterns of meaning (themes) across a data set" (pg. 57). In our case, our data set was composed of results from the micro-experiments from which the BTC practices were developed. Our process for thematic analysis was therefore to consider each of the BTC practices and code them for their underlying principles.

When codes seemed too similar, they were confounded into a single code, and when codes were too broad, they were broken into more specific principles. Practices could have more than one code, but the codes themselves were compared and contrasted with each other until they were deemed unique. For example, the BTC practice of using vertical non-permanent surfaces could be coded as both guided by an effort to *decentralize control* as well as to *mobilize knowledge*, and these denote different principles of practice.

Both authors engaged in this process individually and then compared their results, adapting codes as necessary. This was done iteratively until the smallest possible set of principles of practice emerged that spanned all of the BTC practices.

Results

Through this process, a set of six *principles of practice* emerged that together underpin all the BTC practices. These six principles are listed in Table 1 below, along with the BTC practices they emerged from, and are briefly outlined in what follows.

Table 1. Principles of Practice and the BTC Practices they emerged from.

Principle of Practice	BTC Practices
Decentralize control	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
Give them something to think about	1, 5, 6, 7, 9, 10
Do first	5, 6, 9, 10, 11, 12, 13
Diversity is a strength	2, 6, 9, 10
Mobilize knowledge	2, 3, 4, 5, 7, 8, 9, 10, 11
Assessment as communication	7, 8, 12, 13, 14

Decentralizing control is about the teacher relinquishing control over what students do and how students think. It is therefore also about how authority (in the sense of authorship) is distributed in the learning environment. Contrary to a traditional classroom, in which authority and control are centralized to the teacher, the BTC framework positions the teacher away from the center, granting more power to students to share their ideas and to have capacity in the space. Decentralizing control (and by consequence, distributing authority) in the learning space permeates and underpins almost all the BTC practices. This includes how the environment is set up (e.g., a de-fronted room), how the teacher acts in the space (e.g., how questions are answered), and how students act in the space (e.g., student autonomy). Although the teacher is a central organizer, their aims are (ironically) to step away from the center of organization, and to give value and voice to members of the learning space.

Giving them something to think about is about prioritizing meaningful content that is rich in ideational connections and that offers multiple access points for thinking. It is the guiding principle for what kinds of tasks are selected for a BTC environment as well as how the tasks are given, how questions are answered, how hints and extensions are used, how learning is consolidated, and what homework looks like. In every step where content is introduced or discussed, students ultimately need to have something to think about. If we show students how to do something before they have had a chance to attempt it themselves, we end up with students who mimic procedures without thinking about why they are doing them or how they could adapt them. Therefore, in all the micro- and macro-moves in a BTC environment, our guiding principle is to give students something to think about and to keep them thinking. That is the essence of what the BTC practices were designed to achieve.

Doing first is the foundation of prioritizing learner thinking over *I do—we do—you do*. It guides how we give tasks, how we use hints and extensions, how we answer questions, how students take notes, and how we consolidate the ideas after a period of activity. It also applies to how we evaluate what we value through the co-construction of rubrics after a period of doing, and in self-assessment. In a BTC environment, we engage in action first and then use approaches to consolidate ideas from the environment where activity occurred. This is in stark contrast to traditional teaching methods in which talking about how to do something happens first and is followed by doing what was 'instructed'. BTC practices are designed to reverse this order so that what learners do is rich in thinking, and therefore, not conducive to mimicking. This principle is closely related to *giving them something to think about*, but is more specific to the order in which things occur in a BTC environment.

Diversity is a strength is a perspective that contrasts traditional aims of conformity. Instead, diversity is seen as important for valuing and amplifying learner thinking. This principle influences how both macro- and micro-moves occur in a BTC environment and has to do with both diversity among agents and within their ideas. At the macro-move level, decisions about using visibly random groupings and their size are influenced by this view. Knowing that diversity is necessary in a group for proper group functioning means that when we choose group size, we make sure enough of the diversity can live in that group of agents. At the micro-move level, how we answer questions in groups and how we use hints and extensions are both highly influenced by the view of maximizing the diversity so that new thinking can prevail. It also pertains to consolidating learning, where we emphasize the diversity among ideas in the room. In a broader sense, diversity is seen as necessary for enriching a thinking space and acts as a mindset for a teacher when making decisions.

Mobilizing knowledge is an essential component of how a BTC environment functions. When thinking is at the forefront, any knowledge that is inert needs to be mobilized. This can happen at the individual level, at the small group level, or at the large group level. At the individual level, how we answer questions and how we use hints and extensions aids in pulling ideas out of individuals and possibly, in small groups. That is, it helps ideas become reified into a visible artefact. At the larger group level, how we foster autonomy helps with mobilizing knowledge between groups. The choice of using vertical non-permanent surfaces and visibly random groups in a defronted room aids in mobilizing knowledge since inert ideas are reified on vertical non-permanent surfaces and can then be moved around the room through micro-facilitation strategies. The facilitator can bring groups together or pass on ideas between groups through minor moves such as 'look over there'. Further mobilizing of knowledge occurs during consolidation, where the facilitator draws together the ideas from around the room and weaves them into a meaningful narrative. Mobilizing knowledge continues into the notetaking process where students mobilize the consolidated knowledge in their own personalized ways, and where they find ways to try it out on 'check your understanding' questions. Mobilizing knowledge is therefore the backbone of the entire knowledge cycle in a BTC environment from individual learners to the grand collective of learners and back to individual learners.

Assessment as communication is a mindset that influences how assessment occurs in a BTC environment and what counts as assessment. It reframes the traditional approach of using assessment to control behaviour and to rank learner abilities. Instead, it presupposes a feedback cycle that occurs in how formative assessment, summative assessment, and reporting out are conducted as well as how homework is framed. At the more granular level, assessment as communication also informs how hints and extensions are used to prompt students to think in different ways, thus assessing and communicating at the same time. Rather than making assessment a distinct event, it is occurring continuously as a form of communication in the learning process.

Teaching Online: From Theory to a New Practice

When the COVID-19 pandemic hit and all teaching was forced to move online, adaptations to practice needed to be made. This was inevitably challenging since many of the necessary aspects of a BTC environment, such as 'using vertical non-permanent surfaces' and 'defronting the room' were no longer viable in the online environment. Instead, digital applications dictated the possibilities for what adaptations to practice could look like. In our case, this involved the use of the Zoom conferencing platform to host synchronous online sessions, with the use of hyperlinks to other tools as needed. While there were some other similar platforms available at the time, this platform was chosen based on its feasibility in terms of cost, availability, and ease of use for all participants. Therefore, all the adaptations to practice we address in this paper are based on the possibilities and limitations of the Zoom conferencing platform. Platforms like Zoom typically support institutionally normative practices such as that of having a central authority deliver content to a set of listeners. While some collaborative tools are available (e.g., breakout rooms), limitations on what participants can see or hear still exist and are affected by the various technologies they have access to in their remote sites. These various constraints create new dilemmas around teaching design choices for which guideposts for adapting practice become necessary. To this end, the principles developed in this study became instrumental for creating an effective BTC environment online.

To illustrate how these principles of practice were used to tailor adaptations to the BTC practices for a fully online context, we (the authors) use examples from our own work as educators implementing these adaptations. During the 2020-2021 academic year when teaching moved into a fully online context, we were both working with both pre-service and in-service mathematics teachers in tertiary and professional development settings in various locations in Canada. This included semester-based pre-service teacher education courses at our home institutions as well as single session or series-based workshops for in-service teachers across Canada as part of their professional development. Prior to the pandemic, we had each been using the BTC practices with these groups of adults regularly to engage them in mathematical tasks and reflect on the teaching and learning of mathematics. These groups of adults ranged widely in age and teaching experience, as well as in their mathematical background. Group sizes ranged from as small as 16 in the first author's pre-service secondary mathematics methods course, to more than 100 in the second author's in-service teacher workshop sessions. And session meeting times typically ranged from 2.5 to 6 hours. Notwithstanding, our teaching now could only be conducted via online meetings, which we did via synchronous Zoom meetings, maintaining similar meeting frequencies but now cutting down meetings to about half as long as in pre-pandemic conditions.

In what follows, we discuss how three of the six principles manifested within the online environment: *decentralized control*, *diversity is a strength*, and *mobilizing knowledge*. While the examples we use are drawn from the pre-service and in-service mathematics teachers in our tertiary and professional development settings, our focus remains on our own development of practice. Our purpose in revealing these examples is to support how our teaching decisions were impacted by the principles arising from our analysis of the face-to-face BTC practices; that is, to illustrate how the principles can be harnessed as a theory *for* teaching.

Decentralized control

While almost all the BTC practices decentralize control in some way, the use of vertical non-permanent surfaces is perhaps most notable. In a face-to-face thinking classroom, students use vertical non-permanent surfaces (e.g., whiteboards) to notate their ideas while working in visibly randomized groups. Using such a medium was found to improve students' time to task, time to first mathematical notation, amount of discussion, eagerness to start, participation, persistence, and non-linearity of work (Liljedahl, 2019, 2020). Having students write on vertical non-permanent surfaces also grants them power to erase their work if they so choose. Instead of prioritizing ideas originating from the teacher, learner's ideas are given value in such a space, and can later serve as launch points for consolidation of learning.

When moving into the fully online environment via synchronous Zoom meetings, decentralizing control became more challenging since online meeting platforms such as Zoom are built for speakers presenting to 'the masses', and its tools are centered on showcasing the ideas of one central authority rather than providing opportunities for distributed authority among the agents. That is, it is very well designed for a presentation, but more limited when it comes to supporting collaboration. While making visibly randomized groups became easier with the use of the randomization option when creating breakout rooms in Zoom, decentralization of thinking through vertical non-permanent surfaces became more challenging.

To this end, we employed the use of an external tool that participants could follow a hyperlink to that allowed them some of the features vertical non-permanent surfaces would offer. Namely, we chose to use Google's Jamboard, a web application that allows a number of users to access and edit the document synchronously, with a variety of mediums for notations, and a slide deck organization option for easy access. While Zoom breakout rooms each have their own whiteboard options with some similar tools, these were at the time very limited (e.g., the eraser only would erase the whole screen not parts of it) and they were only visible to the participants in that group. As such, the Zoom whiteboard option offered limited capacity to act as vertical non-permanent surfaces do in a face-to-face setting. Moreover, our principle of decentralizing control shed light on the necessity of having a non-permanent work space for participants that could promote the notation of ideas during a group work task with opportunities for distributing control to participants. In Jamboard, they could create new pages, add new images or drawings, and could refer to various pieces of information that were preloaded into the slides (such as task information). This was not all as easy within Zoom itself.

For example, the Jamboard in Figure 1 below was used as a writing space for groups working on a mathematical task (in this case, a numbers and operations puzzle) in a session with in-service mathematics teachers.

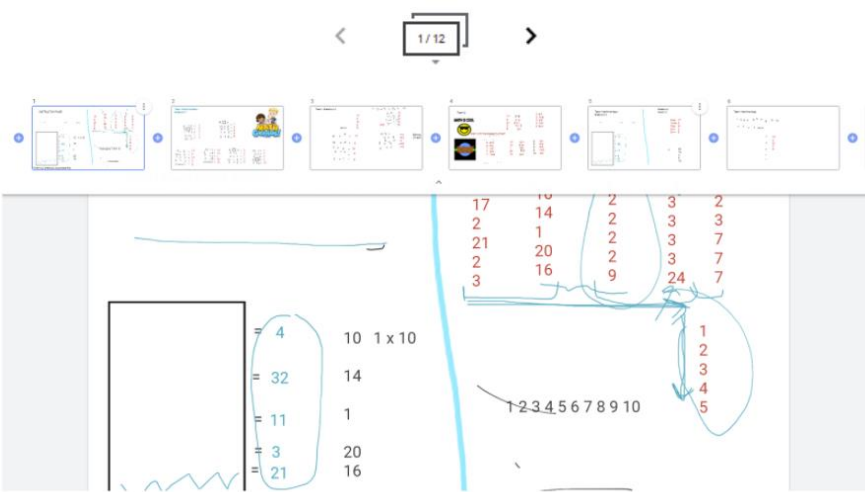

Figure 1. Example of multiple pages of different groups' working spaces on a Jamboard.

As may be seen, each group had their own workspace. As they worked, they could communicate about their ideas verbally via their Zoom breakout room's audio line. And they could add pages, refer to information on other pages, and easily erase or markup as needed. This was also all visible to the teacher, but the teacher was decentralized. Our awareness of the importance of decentralized control in this case supported our choice to use the Jamboard tool.

Diversity is a strength

However, merely implementing the use of a Jamboard did not necessitate a synergy of ideas among participants. When control is distributed, the ideational landscape hinges on having enough diversity in each group of agents (Larsen & Liljedahl, 2017). This means that we had to be careful with how we formed groups beyond that of making sure they were visibly randomized. The principle of seeing diversity as a strength guided us to reconsider group size.

In face-to-face thinking classrooms, research showed the ideal visibly random group size was three (with some twos): having only two students in a group risked the possibility of not having enough diversity among them to develop ideas for the task, while four students in a group risked the possibility of having so much diversity that the stability of the group would be compromised (Liljedahl 2016, 2020). With frequent and visibly randomized groupings of three, students were willing to offer an idea, irrespective of whether they believed their idea would lead to a solution, and students were entering into their collaborative groups willing to think rather than just follow. Students also had opportunities to show various capacities within each grouping, and therefore, could take on various roles each time they were in a new group. The diversity in each grouping easily came to surface since face-to-face settings allow for communication not only via verbal or digital markup tools, but also in gestural ways.

When we shifted to online teaching, the use of frequent random groupings seemed to create silence more so than thinking. Cameras or microphones would be turned off and the technological barriers would be used as a legitimate way to not participate. We believe this had to do with the communication tools available in the online setting. The incredible anonymity in the online setting grants power to students to choose to engage or not. Whereas in a face-to-face thinking classroom setting, students moved quickly into their groupings and promptly began communicating in at least gestural ways, students in an online setting often chose to disengage, cutting out the informal communication that would normally occur haphazardly. And if students end up being placed in a group with others who act this way, even those who would engage more passively are now left out of any engagement at all since they cannot see into other groups' work easily and cannot see what those hidden behind their cameras are trying. This left a smaller number of students in each grouping who were potentially willing to take the active move to communicate with the others they were grouped with.

The 'diversity is a strength' principle helped us identify that the issue with the transfer of this practice to an online environment had to do with the lack of diversity being mobilized in the online setting, leading to it being a diversity depleted space. Therefore, we needed to actively compensate for this tendency of depletion. We found we could compensate for this lack of diversity in two ways. First, we needed to increase the size of the groupings to five or six students rather than three. We found that doing this in the online environment created a similar synergy as when grouping in threes in a face-to-face environment. Doing this increased the number of those willing to engage in each group. As participants became more comfortable with the environment and were more likely to share, the grouping size could be gradually reduced. Second, we understood that sometimes participants needed to work on a task on their own prior to entering the online collaborative environment. This amplified the diversity of ideas in the group enough that when they were placed in the online grouping, they had things to discuss and were not entering empty handed. When we did this in a face-to-face thinking classroom, it created too much diversity for the groups to function cohesively. But in online environments, providing this individual thinking time prior to collaboration seemed to boost the diversity to a minimum functional level.

Mobilizing knowledge

However, the diversity of ideas not only needed to exist in a group, but also needed to be mobilized within and across groups in order for further ideational development to occur. In a face-to-face thinking classroom, knowledge moves between students and groups both through passive and active means (Liljedahl, 2020). Passive knowledge mobility occurs when a participant or a group casually looks to another group's workspace to glean a hint or an extension. Active knowledge mobility occurs when either the teacher pairs groups or groups pair themselves to have more active discussions about specific ideas.

With this in mind, Jamboard was chosen as a participant workspace for the online environment due to its capabilities not only of serving as a digital proxy for a vertical whiteboard, but for its possibilities for knowledge mobility. With its many frames, one Jamboard can accommodate several groups with the possibility of groups being able to easily navigate between frames to see what others are doing (see Figure 2 below).

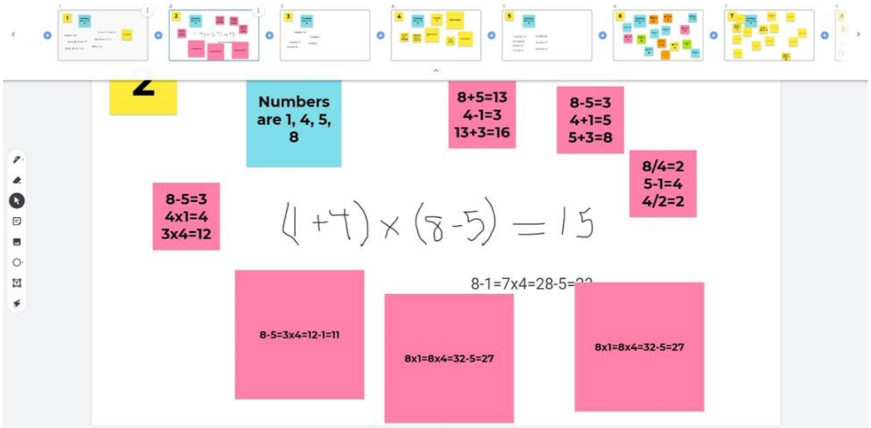

Figure 2. Example of a Jamboard with many group pages.

Jamboards allowed us to afford passive knowledge mobility, but only partially. We found early on in our work in online environments that despite the fact that groups had passive access to other groups' work, they did not make use of this in the same way as in in-person thinking classrooms. This is because navigating between frames in a Jamboard takes a certain active effort. Rather than being able to casually move their gaze from their own vertical whiteboard to another group's board and back again, they needed to click into a frame in order to see it. This, it turns out, was not the same as flipping between frames on a Jamboard. For one, it was more active than the casual glance over the shoulder. In addition, it made it very difficult to look back and forth between frames. We needed to find a way to recreate the casual over the shoulder look that is such a powerful medium for knowledge mobility in the face-to-face thinking classroom, and thus was born the *knowledgefeed*.

A knowledgefeed is a collaborative GoogleDoc that a teacher sets up for students to keep open on their desktop while working in certain settings and is populated with the kinds of things that students would see on VNPSs in a face-to-face classroom. This includes everything from the task at hand, to hints, extensions, and (pictures of) student work (see Figure 3 for an example).

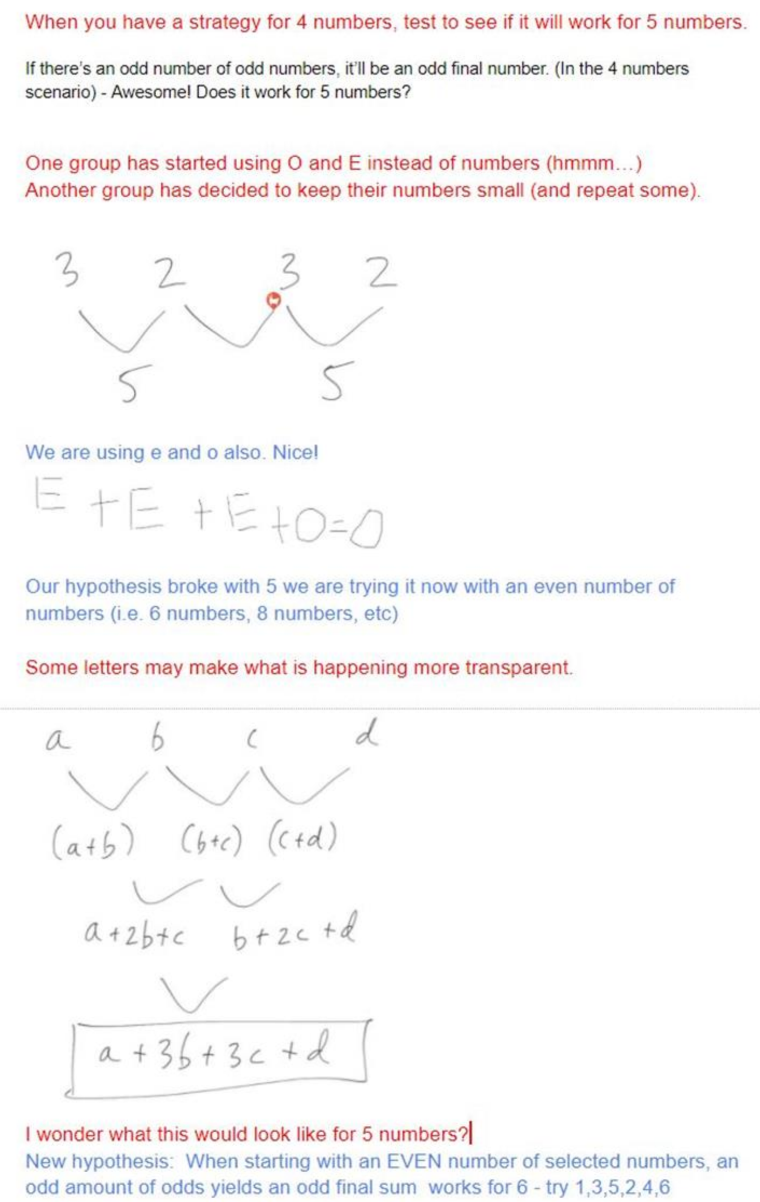

Figure 3. Example of a knowledgefeed for a problem-solving task in a professional development setting.

When we first started to use a knowledgefeed in our online settings, it was something that only we, as the instructors, posted to. After a while, however, it became apparent that participants' voices and images were missing from this feed. Yes, we were adding screen captures from groups' Jamboards, but we were choosing what to add as a way to focus the thinking of the groups. But this is not an accurate analogue of the thinking classroom. In an effort to create a proxy for knowledge mobility we were re-centralizing the control. So, we opened the knowledgefeed to allow students and groups to populate the space with their images, ideas, and questions of their own (see the blue text in figure 3). As they populated it, they also serendipitously encountered other ideas, which they could then bring back into their groups to continue working on their task (e.g., pursuing extensions).

Although allowing participants to contribute made the feed somewhat chaotic, it more accurately approximated the non-linear way in which knowledge moved around in a face-to-face thinking classroom. In this way, the knowledgefeed was a solution created in response to the need for mobilizing knowledge in a way that continued to decentralize control and amplify diversity. It illustrates another choice we made based on the principles identified from our thematic analysis.

Conclusions

Teaching practice requires many micro and macro-level decisions and can require swift adaptations when contexts change. While there is no such thing as a theory *of* teaching to guide necessary adaptations, theories *for* teaching are viable in this regard. In this paper, we explored how a set of principles of practice could be drawn from the BTC teaching practices that had already been established, reified, and validated in their effectiveness in face-to-face settings. These principles of practice served as a theory *for* teaching and helped guide our understanding of which aspects of the face-to-face BTC practices we needed to retain and which we needed to adapt when moving to a fully online teaching setting. By means of a thematic analysis, we moved from practice to theory, which then allowed us to move back to practice in a new setting (i.e., a fully online context). The implications of this are far-reaching.

Of primary importance, we now have a set of six principles of practice that have emerged from the BTC framework for enhancing learning. These principles can now be used to adapt BTC practices to other settings (e.g., online synchronous vs. asynchronous, hybrid synchronous and asynchronous, etc.). By taking into consideration these principles, we can justify our micro- and macro-decisions whenever we face new constraints. While the BTC practices give direction on what to do specifically, the principles give perspective on what is important. In our case, this is to decentralize control, give students something to think about, to do first, to remember that diversity is a strength, to mobilize knowledge, and to treat assessment as communication.

On a broader scale, in going through this process, we have developed an approach for tailoring a theory *for* teaching out of one's own current practice. By using a thematic analysis on the practices used in one context to emerge principles that guide these practices, any educator may then transfer their teaching practices to novel contexts with a more in-depth understanding of their own teaching decisions. That is, the adaptations to teaching practice can be justified and targeted rather than decided on at random. In our case, to determine how best to facilitate mathematics problem solving in adult tertiary and professional development online settings, we could have proceeded in many ways. For instance, we could have succumbed to institutional norms of following an *I do - we do - you do* model because of software constraints. Or, we could have aimed to mimic the BTC face-to-face practice as exactly as possible (e.g., keeping the same grouping sizes) while guessing at how to replicate practices that were no longer possible (e.g., defronting the room). However, emerging principles of practice before importing them into a new setting helped us, as educators, to be able to justify our decisions, to adapt intentionally, and to be more confident with implementation of the adaptations.

Given the variety of settings found in adult mathematics education, traversing teaching contexts can occur. This approach of reifying principles of practice may be a way to guide and justify adaptations. And given that the principles of practice we developed in this paper emerged from practices designed to challenge institutional norms, these principles may serve to break patterns of cultural reproduction in settings that remind adult learners of their subjectivities towards mathematics as rooted in their past experiences with learning mathematics.

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